32. *An Approximate Positive Part of Essentially Self-Adjoint Pseudo-Differential Operators. II*

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§ 1. **Introduction.** Let $a(x, \xi)$ be a real valued symbol function belonging to the class $S^1_{10}(\mathrm{R}^n)$ of Hörmander [2], that is, for any pair of multi-indices α and β, we have

$$\sup (1+|\xi|^2)^{(|\beta|-1)/2}|D^\alpha_x D^\beta_\xi a(x, \xi)|<\infty,$$

where we used usual multi-index notation. As the continuation of the previous note [1], we treat the Weyl quantization $a^w(x, D)$ of it, which is defined as

$$a^w(x, D)u(x)=\left(\frac{1}{2\pi}\right)^n \int_{\mathrm{R}^n}\int_{\mathrm{R}^n} a\left(\frac{x+y}{2}, \xi\right) e^{i(x-y)\cdot\xi}u(y)\, dy\, d\xi. \tag{1.1}$$

Cf. Weyl [6], Voros [5], and Hörmander [3].

Let $(\ ,\ )$ and $\|\ \|$ denote the inner product and the norm, respectively, in $L^2(\mathrm{R}^n)$. In the previous note, we reported the following

Theorem 1. *Let ε be an arbitrary small positive number. Then, using the symbol function $a(x, \xi)$, we can construct three bounded linear operators π^+, π^- and R in $L^2(\mathrm{R}^n)$ with the following properties:*

1) *Both π^+ and π^- are non-negative symmetric operators.*

2) *There exists a positive constant C such that we have*

$$Re(\pi^+ a^w(x, D)u, u)\geq -C\|u\|^2 \tag{1.2}$$

$$-Re(\pi^- a^w(x, D)u, u)\geq -C\|u\|^2 \tag{1.3}$$

for any $u \in \mathcal{S}(\mathrm{R}^n)$.

3) $$\pi^+ +\pi^- =I+R, \qquad \|R\|<\varepsilon, \qquad \text{and}$$
$$\|a^w(x, D)R\|<\infty, \qquad \|R\, a^w(x, D)\|<\infty.$$

Let

$$\mathfrak{C}^+(a)=\{(x, \xi)|a(x, \xi)\geq 0\}$$

and

$$\mathfrak{C}^-(a)=\{(x, \xi)|a(x, \xi)\leq 0\}.$$

We call $\mathfrak{C}^0(a)=\mathfrak{C}^+(a)\cap\mathfrak{C}^-(a)$ the characteristic set of a. The aim of this note is to show the following

Theorem 2. *Let $a(x, \xi)$ and $p(x, \xi)$ be two real valued functions in $S^1_{10}(\mathrm{R}^n)$. Suppose the following two conditions hold:*

(A) $\mathfrak{C}^+(a)\subset\mathfrak{C}^+(p), \qquad \mathfrak{C}^-(a)\subset\mathfrak{C}^-(p).$

(B) *There exists a positive constant C such that*

$$|\mathrm{grad}_x p(x, \xi)|\leq C|\mathrm{grad}_x a(x, \xi)| \tag{1.4}$$

$$|\mathrm{grad}_\xi p(x, \xi)|\leq C|\mathrm{grad}_\xi a(x, \xi)| \tag{1.5}$$