27. *Zeta Functions in Several Variables Associated with Prehomogeneous Vector Spaces. II*[*)]

A Convergence Criterion

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(Communicated by Shokichi IYANAGA, M. J. A., Feb. 12, 1981)

This article is a continuation of [2]. Here we present a sufficient condition of the convergence of zeta functions introduced in the previous paper. We keep the notation and the assumptions in [2] except (A.3).

5. Let H be the identity component (in the Zariski topology) of the group

$$\{g \in G;\ \chi(g)=1 \quad \text{for all } \chi \in X_\rho(G)\}.$$

We consider the following four conditions:

(I) *For any* $t=(t_1, \cdots, t_n) \in (C^\times)^n$, H *acts transitively on*

$$V(t)=\{x \in V-S;\ P_i(x)=t_i\ (1 \leq i \leq n)\}.$$

(S) *The group*

$$H_x=\{g \in H;\ \rho(g)x=x\}$$

is a connected semi-simple algebraic group for any $x \in V-S$.

(W) *For any* $x \in V_Q - S_Q$, *the Tamagawa number of* H_x *does not exceed some positive constant independent of* x.

(H) *For any* $x \in V_Q - S_Q$ *and for any inner* $\boldsymbol{Q}$*-form* A *of* H_x, *the canonical mapping*

$$H^1(\boldsymbol{Q}, \tilde{A}) \longrightarrow \prod_v H^1(\boldsymbol{Q}_v, \tilde{A})$$

is bijective where $\tilde{A}$ *is the universal covering group of* A *defined over* $\boldsymbol{Q}$ *and the product is over all places of* $\boldsymbol{Q}$.

Theorem 4. *If* (G, ρ, V) *satisfies the conditions* (A.1), (I), (S), (W), *and* (H), *then* (G, ρ^*, V^*) *also satisfies the conditions* (I), (S), (W) *and* (H). *Moreover the integrals* $Z(f, L; s)(f \in \mathcal{S}(V_{\boldsymbol{R}}))$ *and* $Z^*(f^*, L^*; s)(f^* \in \mathcal{S}(V^*_{\boldsymbol{R}}))$ *are absolutely convergent when* $\mathrm{Re}\, s_1, \cdots, \mathrm{Re}\, s_n$ *are sufficiently large.*

Theorem 5. *Further assume that every* $\boldsymbol{Q}$*-irreducible component of* S *is absolutely irreducible. Then* $Z(f, L; s)$ *(resp.* $Z^*(f^*, L^*; s)$*) is absolutely convergent for* $\mathrm{Re}\, s_1 > \delta_1, \cdots, \mathrm{Re}\, s_n > \delta_n$ *(resp.* $\mathrm{Re}\, s_1 > \delta_1^*, \cdots, \mathrm{Re}\, s_n > \delta_n^*$*).*

Remarks. (1) The condition (S) implies the condition (A.2).

*) Supported by the Grant in Aid for Scientific Research of the Ministry of Education No. 574050.