110. *Modular Representations of p-Groups with Regular Rings of Invariants*

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§1. **Introduction.** Let V be an n-dimensional vector space over a field k of characteristic p and G a finite subgroup of $GL(V)$. Then G acts linearly on the symmetric algebra R of V. We denote by R^G the subring of R consisting of all invariant polynomials under this action of G. The following theorem is well known.

(1.1) **Theorem** (Chevalley-Serre, cf. [2], [3], [5]). *Suppose that* $p=0$ *or* $(|G|, p)=1$. *Then* R^G *is a polynomial ring if and only if* G *is generated by pseudo-reflections in* $GL(V)$ *(an element* σ *of* $GL(V)$ *is said to be a pseudo-reflection if* rank $(\sigma-1)\leqq 1$).

Now we assume that $p>0$ and that the order of G is divisible by p. Serre obtained a necessary condition for R^G to be a polynomial ring as follows.

(1.2) **Theorem** (Serre, cf. [2], [5]). *If* R^G *is a polynomial ring, then* G *is generated by pseudo-reflections.*

However the converse of (1.2) is not always true. For example $R^{O_n(F_q)}$ ($n\geqq 4$, p odd) are not polynomial rings, where $O_n(F_q)$ are orthogonal subgroups of $GL(V)$ of dimension n defined over the subfield F_q of k consisting of q elements.

Hereafter we suppose that k is the prime field of characteristic $p(>0)$ and that G is a p-subgroup of $GL(V)$.

The purpose of this note is to announce our results on rings of invariants of p-groups. We can completely determine p-groups G such that R^G are polynomial rings. The main result is

(1.3) **Theorem.** *The following statements on a pair of* V *and* G *are equivalent*:

(1) R^G *is a polynomial ring.*

(2) *There is a* k*-basis* $\{X_1, \cdots, X_n\}$ *of* V *with the equality*

$$\prod_{i=1}^{n} |GX_i| = |G|$$

such that all $\oplus_{i=1}^{j} kX_i$ $(1\leqq j\leqq n)$ *are* kG*-submodules of* V.

In [1] it has been shown that if G is a p-Sylow subgroup of $GL(V)$, R^G is a polynomial ring.

§2. **Preliminaries.** We need some lemmas on invariant subrings of polynomial rings: