# 96. *Calculus on Gaussian White Noise. II*

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We are going to reformulate the works of Hida [1], [2] to establish a calculus on generalized Brownian functionals which we call Hida calculus.

In Part I [11], we have prepared fundamental tools. By using them, we will discuss on generalized random variables, annihilation operators $\partial_t$, creation operators $\partial_t^*$, multiplications $x(t)\cdot$ and so forth.

**§ 5. Generalized random variables.** As assumed in § 4 of Part I [11], let T be a separable metrizable space with a $\sigma$-finite Borel measure $\nu$ and put $E_0=L^2(T,\nu)$. Let $\mathcal{E}$ be a dense subset of $E_0$ which has a consistent sequence of inner products $\{(\xi,\eta)_p\,;\,p\geq 0\}$ such that

(5.1) $\qquad (\xi,\xi)_p\leq\rho(\xi,\xi)_{p+1},\quad$ for $p\geq 0\quad$ with $\quad\rho,\ 0<\rho<1.$

Let $E_p$ be the completion of $\mathcal{E}$ by the norm $\|\ \|_p$ and $E_{-p}=E_p^*$ with $(\xi,\eta)_{-p}$ be the dual of $E_p$. Suppose that $\mathcal{E}$ is identical to the projective limit $E_\infty$ of $E_p$. Then the dual $\mathcal{E}^*$ is the inductive limit $E_{-\infty}$ of $E_{-p}$. Throughout this note we assume that the injection $\iota_{0,1}$ from $E_1$ to $E_0$ is *traceable*; that is, $\delta_t:\xi\mapsto\xi(t)$ belongs to $E_{-1}$ and the mapping $t\in T\to\delta_t\in E_{-1}$ is continuous, and assume that $\|\delta\|^2\equiv\int_T\|\delta_t\|_{-1}^2\,d\nu(t)<\infty$. Then by Lemma 4.2, the injection $\iota_{0,1}$ is a Hilbert-Schmidt operator. Therefore, by Gelfand-Minlos-Sazanov's theorem, we have

**Theorem 5.1.** *There exists a probability measure $\mu$ on $\mathcal{E}^*$ such that*

$$\int_{\mathcal{E}^*} e^{i\langle x,\xi\rangle}d\mu(x)=\exp\left[-\frac{1}{2}\|\xi\|_0^2\right],\qquad for\ \xi\in\mathcal{E}.$$

**Definition 5.2.** The measure $\mu$ on $\mathcal{E}^*$ is called a *measure of Gaussian white noise.* The $L^2$-space $L^2(\mathcal{E}^*,\mu)$ is denoted by $(L^2)$, simply.

It is well known that the measure $\mu$ is quasi-invariant under the shift $x\to x-\xi$ for $\xi\in\mathcal{E}$ and that

$$\frac{d\mu(x-\xi)}{d\mu(x)}=\exp\left[\langle x,\xi\rangle-\frac{1}{2}\|\xi\|_0^2\right]\in L^q(\mathcal{E}^*,\mu)\tag{5.2}$$

for $q\geq 1$ [7]. With the result, we can define a transformation $\mathcal{S}$ by

$$(\mathcal{S}\varphi)(\xi)=\int_{\mathcal{E}^*}\varphi(x+\xi)d\mu(x),\quad \xi\in\mathcal{E},\quad \varphi\in L^q(\mathcal{E}^*,\mu),\quad 1<q<\infty.\tag{5.3}$$

**Remark 5.3.** By (5.2) and (5.3), $(\mathcal{S}\varphi)\,(\lambda\xi)$ can be extended to an entire function of $\lambda$ as follows;