94. On Determinants of Cartan Matrices of p-Blocks

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1. Introduction. Let B be a p-block of a finite group with defect group D, and C_B the Cartan matrix of B. Then it is known that $\det C_B \geq |D|$. In [6] we showed that the equality holds in the above under some assumption. The purpose of this note is to extend this result.

Notation. Let G be a finite group with order divisible by a fixed prime p and $\mathfrak{p}$ a fixed prime divisor of p in the ring $\boldsymbol{Z}[\varepsilon]$, where ε is a primitive $|G|$-th root of 1. We denote by F the residue class field $\boldsymbol{Z}[\varepsilon]/\mathfrak{p}$, by FG the group algebra of G over F, and by $Z(FG)$ the center of FG. If B is a block of G, we denote by C_B the Cartan matrix of B, by $D(B)$ a defect group of B, and by $l(B)$ the number of irreducible modular characters in B. If Q is a p-subgroup of $G, m_B(Q)$ denotes the number of p-regular (conjugate) classes of G associated with B which have Q as a defect group. (For selection of sets of conjugate classes for the blocks, see Brauer [1], [2], [4], Osima [8], and Iizuka [7].) We denote by $S(B)$ the set of subsections $s=(\pi, b)$ associated with B which are different from $1=(1, B)$. (For a subsection, see Brauer [3].) For brevity we write $C(X)$ and $N(X)$ instead of $C_G(X)$ and $N_G(X)$ for a subset X of G respectively. If K is a conjugate class of G, we denote by $\hat{K}$ the class sum of K in the group algebra FG.

The main result of this note is the following

Theorem. *Let B be a block of G.*

(i) *For a proper subgroup $Q\neq 1$ of $D(B)$, if $m_B(Q)\neq 0$, then $m_b(Q)\neq 0$ for some $s=(\pi, b)\in S(B)$ such that $D(b)$ contains Q as a proper subgroup.*

(ii) *If* $\det C_b=|D(b)|$ *for any $s=(\pi, b)\in S(B)$, then* $\det C_B=|D(B)|$.

Next corollary is an immediate consequence of Theorem, (ii).

Corollary 1. *Let B be a block of G with defect group D. Suppose that $l(b)=1$ for any $s=(\pi, b)\in S(B)$. Then* $\det C_B=|D|$.

As a special case we have the following

Corollary 2 (Fujii [6]). *Let B be a block of G with defect group D. Suppose that the centralizer in G of any element of order p of D is p-nilpotent. Then* $\det C_B=|D|$.

Remark. The first part of Theorem still holds even if we denote by $m_B(Q)$ the number of conjugate classes of G associated with B which