88. *Calculus on Gaussian White Noise. I*

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§ 1. **Introduction.** Recently, Hida has introduced *generalized Brownian functionals* to discuss the analysis on the L^2-space (L^2) built on the measure space of white noise $\dot{B}(t)$. The idea of Hida's analysis is to take $\{\dot{B}(t)\}$ to be the system of the variables of Brownian functionals, so that we are led to introduce multiplication operators $\dot{B}(t)$ and the partial differential operators $\partial/\partial\dot{B}(t)$ as well as renormalization of functions of the $\dot{B}(t)$'s [1, 2]. We will give, in this series of notes Parts I–V, a systematic treatment of his analysis and establish formulae which would make easier to apply his theory.

We will discuss, in Part I, a general theory on Fock spaces and Hilbert spaces of non-linear functionals of special types, which is a slight modification of the works of Segal [3], [4] and of Hida-Ikeda [5].

In Part II, the L^2-space $(L^2)=L^2(\mathcal{E}^*, \mu)$ will be discussed, where $\mathcal{E}\subset E_0\subset\mathcal{E}^*$ is a Gelfand triplet and μ is the measure of Gaussian white noise on $\mathcal{E}^*$. With the help of transformation $\mathcal{S}$,

$$(\mathcal{S}\varphi)(\xi)=\int_{\mathcal{E}^*}\varphi(x+\xi)d\mu(x),\ \xi\in\mathcal{E},\ \varphi\in(L^2),$$

we can apply the analysis established in Part I. We will treat operators $\partial/\partial x(t)$, $(\partial/\partial x(t))^*$, $x(t)\cdot=\partial/\partial x(t)+(\partial/\partial x(t))^*$ and so forth to carry on the proposed analysis of Brownian functionals.

In Part III, we will describe Hida's analysis by our formulation, partly. In Part IV, Laplacians on (L^2) will be discussed. In Part V, we will discuss Hida-Streit's approach to Feynman path integral in line with our formulation.

§ 2. **Triplets of Fock spaces.** Let $(E_0, (\xi, \eta)_0)$ be a separable real Hilbert space and let us identify its dual E_0^* with E_0. Suppose that $\mathcal{E}$ is a dense linear subset of E_0. Let $\{(\ ,\ )_p\,;\,p\geq 0\}$ be a consistent sequence of inner products defined on $\mathcal{E}$ such that

(2.1) $\qquad \|\xi\|_0\leq\rho\,\|\xi\|_1\leq\cdots\leq\rho^p\,\|\xi\|_p\ \cdots,\quad$ with $\quad\rho\in(0,1)$.

Let E_p be the completion of $\mathcal{E}$ in $\|\ \ \|_p$, and $E_{-p}\equiv E_p^*$ be the dual of E_p with the inner product $(\ ,\ )_{-p}$, for $p>0$. Then we have inclusions

$$\cdots\subset E_{p+1}\subset E_p\subset\cdots\subset E_0\subset\cdots\subset E_{-p}\subset E_{-p-1}\cdots.$$

Let E_∞ be the projective limit of the system $\{(E_p, \|\ \ \|_p)\,;\,p\in Z\}$. Suppose that $\mathcal{E}=E_\infty$ as a set and induce the topology by this equality.