9. On Unramified $SL_2(F_4)$ Extensions of an Algebraic Function Field

By Hidenori KATSURADA
Department of Mathematics, Hokkaido University

(Communicated by Shokichi IYANAGA, M. J. A., Jan. 12, 1980)

The purpose of this note is to report some results on the number of unramified $SL_2(F_4)$ extensions of some algebraic function field of characteristic 2. Detailed accounts are stated in [1] and [2].

§0. Main results. Let k be an algebraically closed field of characteristic 2. Let $K=k(x, y)$ be an algebraic function field over k defined by $y^2-y=x^5-\alpha x^3$ ($\alpha \in k$). Let $\tilde{K}$ be the maximum unramified Galois extension of K and let $A_{SL_2(F_4)}$ be the set of $GL_2(k)$ equivalence classes of representations of $\mathrm{Gal}(\tilde{K}/K)$ onto $SL_2(F_4)$. We put

$$B=\left\{\begin{array}{l} (X, Y, Z, \lambda) \in P^2\times A^1;\ X^2Z^2+Y^3Z+(c_4X+Y)X^3=0, \\ \qquad Y^9Z^8+ZX^{16}+c_4^2YX^{16} \\ \qquad\qquad +(X+\alpha^2Y)(Y^8X^8+\alpha^4X^{16})=0, \\ \qquad YX^{16}+(X+\alpha^2Y)(X^{16}\alpha^8+Y^{16})=0, \\ \qquad c_4=\lambda^{16}+\alpha^4\lambda^8+\alpha^2\lambda^2+\lambda,\ Z\neq 0, \\ \qquad \alpha^2Z^2Y+Y^2Zc_4+X^3c_4\neq 0 \end{array}\right\}.$$

Then one of our main results is:

Theorem 1. *There is a* 2:1 *map of* B *onto* $A_{SL_2(F_4)}$.

By making use of this theorem and some other considerations, we can show the following

Theorem 2. $\sharp A_{SL_2(F_4)}=640$ *if* $\alpha=0$,
$=736$ *otherwise.*

Corollary to Theorem 2. *The number of unramified* $SL_2(F_4)$ *extensions of* K *is* 320 *if* $\alpha=0$ *and* 368 *otherwise.*

§1. Representations of $\mathrm{Gal}(\tilde{K}/K)$ into $GL_n(F_q)$. Let K_A be the adele ring of K, let $\mathfrak{O}$ be the integer ring, and let $\mathfrak{U}$ be the unit group of $\mathfrak{O}$. We put $G_n=GL_n(\mathfrak{O})\backslash GL_n(K_A)/GL_n(K)$. Then, the map $GL_n(K_A) \ni (u_{ij})\mapsto(u_{ij}^q) \in GL_n(K_A)$ induces a map $F(q)$ of G_n into itself. We denote by $\mathrm{Rep}(GL_n(F_q))$ the set of $GL_n(k)$ equivalence classes of representations of $\mathrm{Gal}(\tilde{K}/K)$ into $GL_n(F_q)$. Then we have:

Proposition 1.1. *There is a one to one correspondence between the set* $G_n^{F(q)}$ *of* $F(q)$ *fixed points of* G_n *and* $\mathrm{Rep}(GL_n(F_q))$.

For any element R of $GL_n(K_A)$, we denote by $[R]$ the element of G_n whose representative is R.

Corollary to Proposition 1.1. *We put*

$$S_n=\{[R] \in G_n \text{ satisfying } \det R=1\}.$$