84. *Some Examples of Analytic Functionals with Carrier at the Infinity*

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In this note we propose some examples of analytic functionals with carrier at the infinity. In particular, we will give an example of a Fourier hyperfunction with support at the infinity.

We confine ourselves to the one dimensional case and follow the notations in Morimoto [2] and Morimoto-Yoshino [3]. Let $L=A+iK$, $i=\sqrt{-1}$, $A=[a, \infty)$, $K=[-k, k]$ and $k'\in \boldsymbol{R}$. We denote by $Q_b(L\,;k')$ the space of all continuous functions f on L holomorphic in the interior of L which satisfy the following condition:

(1) $$\sup\{|f(\zeta)|\exp(k'\xi)\,;\zeta=\xi+i\eta\in L\}<\infty.$$

Taking the inductive limit following the restriction mappings as $\varepsilon\downarrow 0$ and $\varepsilon'\downarrow 0$, we define the fundamental space

(2) $$Q(L\,;k')=\lim_{\substack{\varepsilon\downarrow 0\\ \varepsilon'\downarrow 0}}\operatorname{ind} Q_b(L_\varepsilon\,;k'+\varepsilon'),$$

where $L_\varepsilon=[a-\varepsilon, \infty)+i[-k-\varepsilon, k+\varepsilon]$. A continuous linear functional S on the space $Q(L\,;k')$ is, by definition, an analytic functional with carrier in L and of exponential type k'. $Q'(L\,;k')$ will denote the dual space of $Q(L\,;k')$. An analytic functional S is said to be with carrier in $\infty+iK$ if $S\in Q'([a,\infty)+iK\,;k')$ for every $a>0$.

We recall three transformations of analytic functionals:

1) The Cauchy transformation of $S\in Q'(L\,;k')$ is defined by the following formula:

(3) $$\check{S}(\tau)=\frac{-1}{2\pi i}\left\langle S_\zeta, \frac{\exp(-(\tau-\zeta)^2)}{\tau-\zeta}\right\rangle.$$

It is known that $\check{S}(\tau)$ is a holomorphic function on $\boldsymbol{C}\backslash L$, satisfying, for any positive numbers ε, r and ε' with $0<\varepsilon<r$,

(4) $$\sup\{|\check{S}(\tau)|\exp(-(k'+\varepsilon')s)\,;\tau=s+it\in L_r\backslash L_\varepsilon\}<\infty$$

and that we have the inversion formula

(5) $$\langle S, f\rangle=-\int_{\partial L_\varepsilon}\check{S}(\tau)f(\tau)d\tau$$

for every $f\in Q(L\,;k')$, where $\varepsilon>0$ is sufficiently small (Theorems 3.2 and 3.3 in Morimoto [2]).

2) The Fourier-Borel transformation $\tilde{S}$ of $S\in Q'(L\,;k')$ is defined by

(6) $$\tilde{S}(z)=\langle S_\zeta, \exp(z\zeta)\rangle.$$

It is known the Fourier-Borel transformation $\mathcal{F}: S\mapsto\tilde{S}$ establishes a