76. *Some Results in the Classification Theory of Compact Complex Manifolds in $\mathcal{C}$*

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0. By definition a compact complex manifold X is in $\mathcal{C}$ if there exist a compact Kähler manifold Z and a surjective meromorphic map $h: Z \to X$ [1]. The purpose of this note is then to report some results on the structure of manifolds in $\mathcal{C}$. Details will appear elsewhere.

In what follows X, Y, Z, etc. always denote compact connected complex manifolds in $\mathcal{C}$. We set $q(X) = \dim H^1(X, O_X)$ and $a(X) =$ the algebraic dimension of X [2]. Let $f: X \to Y$ be a holomorphic map. For any open subset $U \subseteqq Y$ we write $X_U = f^{-1}(U)$ and $f_U = f|_{X_U}$, and write $X_y = f^{-1}(y)$ for $y \in Y$. We call f a *fiber space* if f is proper and surjective and has connected fibers. Suppose that f is a fiber space. Then we set $\dim f = \dim X - \dim Y$, and $q(f) = q(X_y)$ for any smooth fiber X_y. Further any fiber space $f^*: X^* \to Y^*$ which is bimeromorphic to f is called a *bimeromorphic model* of f.

1. Let $f: X \to Y$ be a fiber space and U a Zariski open subset of Y over which f is smooth. For any integer $k \geqq 0$ we set $A_k = \{u \in U; a(X_u) \geqq k\}$.

Proposition 1. *A_k is a union of at most countably many analytic subsets of U whose closures are analytic in Y.*

Let $a(f) = \max\{k; A_k = U\}$, so that $a(X_u) = a(f)$ for 'general' $u \in U$. We call $a(f)$ the *relative algebraic dimension* of f. By Proposition 1 $a(f)$ depends only on the bimeromorphic equivalence class of f. Clearly $0 \leqq a(f) \leqq \dim f$.

Proposition 2. *Let $f: X \to Y$ and U be as above. Then the following conditions are equivalent.* 1) $a(f) = \dim f$, 2) $f_U: X_U \to U$ *is locally Moishezon, and* 3) *there exists a bimeromorphic model $f^*: X^* \to Y^*$ of f which is locally Moishezon.*

Here a morphism $g: X \to Y$ is called *locally Moishezon* if for each $y \in Y$ there exists a neighborhood $y \in V$ such that $g_V: X_V \to V$ is Moishezon, i.e., bimeromorphic over V to a projective morphism.

Definition 1. Let $f: X \to Y$ be a fiber space. Then a *relative algebraic reduction* of f is a commutative diagram

$$\begin{tikzcd} X^* \arrow[rr, "f^*"] \arrow[dr, "g"'] & & Y^* \\ & Z \arrow[ur, "h"'] & \end{tikzcd}$$