7. *Representation Groups of the Group* $Z_{p^n}\times Z_{p^n}$

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Introduction. The dihedral group D_2 and the quaternion group Q_2 of order 8 have the same character table (Feit [1, §§7 and 11]). Generally the two non-abelian groups of order p^3 (p a prime number) have the same character table (Brauer [3, §4]). It is easily shown that these groups are characterized as the representation groups of the product $\boldsymbol{Z}_p\times\boldsymbol{Z}_p$ of cyclic groups of order p.

In this note, we consider the representation groups of $\boldsymbol{Z}_{p^n}\times\boldsymbol{Z}_{p^n}$, the product of cyclic groups of order p^n, and we deal with those complex characters. In §1, we show that there exist two non-isomorphic representation groups of $\boldsymbol{Z}_{p^n}\times\boldsymbol{Z}_{p^n}$ (Theorem 1). When $n\geqq 2$, these groups have not the same character table (§3, Corollary 2), but have the conjugacy classes of the type described in Proposition 1. Their non-linear irreducible characters are constructed by the abelian residue groups of certain normal subgroups (Theorem 2).

1. Generators and relations. Let G be a finite group and $\boldsymbol{C}^*$ the multiplicative group of the complex number field $\boldsymbol{C}$. When G acts trivially on $\boldsymbol{C}^*$, the finite abelian group $H^2(G,\boldsymbol{C}^*)$ is called the Schur multiplier of G. A group H is called a representation group of G when H has a central subgroup A such that 1) $H/A\cong G$, 2) $|A|=|H^2(G,\boldsymbol{C}^*)|$ and 3) A is contained in the commutator subgroup $D(H)$.

Let H be a representation group of $\boldsymbol{Z}_{p^n}\times\boldsymbol{Z}_{p^n}$, where p is a prime number and n is a positive integer. The sequence

$$1\to A\to H\to \boldsymbol{Z}_{p^n}\times\boldsymbol{Z}_{p^n}\to 1$$

is exact, and $A=D(H)$ is contained in the center $Z(H)$ of H. We choose representatives t, r of inverse images of two generators of $\boldsymbol{Z}_{p^n}\times\boldsymbol{Z}_{p^n}$. Then A is the cyclic group generated by the commutator $s=t^{-1}rtr^{-1}$ of order p^n, because $H^2(\boldsymbol{Z}_{p^n}\times\boldsymbol{Z}_{p^n},\boldsymbol{C}^*)\cong\boldsymbol{Z}_{p^n}$ (see Suzuki [2, p. 261]).

Consequently, the elements t, r and s generate H, i.e.,

$$H=\langle t, r, s\rangle \tag{1}$$

and satisfy the relations

$$r^{p^n},\quad t^{p^n}\in\langle s\rangle,\quad s^{p^n}=1 \tag{2}$$

$$ts=st,\quad rs=sr\quad\text{and}\quad t^{-1}rt=rs \tag{3}$$

where p^n is the least positive integer q such that $t^q\in\langle s\rangle$ (this p^n is also the least positive integer q such that $r^q\in\langle s\rangle$). Note that $A=Z(H)$.