63. *On Surfaces of Class VII_0 with Curves*

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§1. Let S be a surface, i.e., a compact complex manifold of complex dimension 2. We write $b_i(S)$ for the i-th Betti number of S. For a divisor D on S, we write D^2 for its self intersection number. A surface S is said to be *of Class VII_0* if S is minimal and $b_1(S)=1$. When a surface S is of Class VII_0, it is well known that any divisor D on S has $D^2 \leqq 0$.

In this note, we shall state theorems on a surface of Class VII_0 which has a divisor D with $D^2=0$. For this purpose, we shall construct surfaces $S_{n,\alpha,t}$ $(n>0,\ 0<|\alpha|<1,\ t\in \boldsymbol{C}^n)$, which satisfy the following conditions:

(1.1) $S_{n,\alpha,t}$ is of Class VII_0,

(1.2) $b_2(S_{n,\alpha,t})=n$,

(1.3) $S_{n,\alpha,t}$ has a connected curve $D_{n,\alpha,t}$ with $D^2_{n,\alpha,t}=0$.

Our main result is the following

Theorem 1. *Let S be a surface of Class VII_0 with $b_2(S)=n>0$. If S has a divisor $D\neq 0$ with $D^2=0$, then S is biholomorphic to $S_{n,\alpha,t}$ for some $0<|\alpha|<1$, $t\in \boldsymbol{C}^n$ and $D=mD_{n,\alpha,t}$ for some integer $m\neq 0$.*

In view of the classification theory of Kodaira on surfaces, Theorem 1 implies

Theorem 2. *Let S be a surface and C be a curve on S. Assume that*

i) *there is a non-constant holomorphic function on $S-C$,*

ii) *the number of compact irreducible curves on $S-C$ is finite.*

Then $S-C$ has a structure of a quasi-projective variety.

To state theorems on deformations of $S_{n,\alpha,t}$, set

$$T_n=\{\alpha\in \boldsymbol{C}\,|\,0<|\alpha|<1\}\times \boldsymbol{C}^n,$$
$$\mathcal{S}_n=\bigcup_{(\alpha,t)\in T_n} S_{n,\alpha,t} \qquad \text{(disjoint union)},$$
$$\mathcal{D}_n=\bigcup_{(\alpha,t)\in T_n} D_{n,\alpha,t} \qquad \text{(disjoint union)},$$
$$\mathcal{A}_n=\mathcal{S}_n-\mathcal{D}_n, \qquad A_{n,\alpha,t}=S_{n,\alpha,t}-D_{n,\alpha,t}.$$

Let $\pi:\mathcal{S}_n\to T_n$ be the projection so that $\pi^{-1}(\alpha,t)=S_{n,\alpha,t}$. Let $\iota: S_{n,\alpha,t}\to\mathcal{S}_n$ be the natural inclusion. Then $\mathcal{S}_n$ has a complex structure such that the projection π is a holomorphic map of maximal rank and the inclusion ι is biholomorphic. Let Θ be the sheaf of germs of holomorphic vector fields on $S_{n,\alpha,t}$, i.e., the sheaf of germs of infinitesimal