59. A Note on Quasilinear Evolution Equations

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§ 1. **Introduction.** In this note we give a generalization of the result of Massey [2] who proved analyticity in t of solutions to quasilinear evolution equations

$$\frac{du}{dt}+A(t,u)u=f(t,u), \qquad 0\leqq t\leqq T, \tag{1.1}$$

$$u(0)=u_0. \tag{1.2}$$

The unknown, u, is a function of t with values in a Banach space X. For fixed t and $v\in X$, the linear operator $-A(t,v)$ is the generator of an analytic semigroup in X and $f(t,v)\in X$. We consider the equation (1.1) under the assumption that the domain $D(A(t,u)^h)$ of $A(t,u)^h$ is independent of t,u for some $h>0$, while Massey discussed it in the case that $D(A(t,u))$ is constant.

In the following $L(X,Y)$ is the space of linear operators from normed space X to normed space Y, and $B(X,Y)$ is the space of bounded linear operators from normed space X to normed space Y. $L(X)=L(X,X)$ and $B(X)=B(X,X)$. $\|\ \|$ will be used for the norm in both X and $B(X)$.

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§ 2. **The main result.** We shall make the following assumptions:

A-1°) $u_0\in D(A_0)$ and $A_0^{-\alpha}$ is a well-defined operator $\in B(X)$ where $A_0\equiv A(0,u_0)$.

A-2°) There exist $h=1/m$, where m is an integer, $m\geqq 2$, $R>0$, $T_0>0$, $\phi_0>0$ and $0\leqq\alpha<h$, such that $A(t,A_0^{-\alpha}w)$ is a well-defined operator $\in L(X)$ for each $t\in\Sigma_0\equiv\{t\in C;|\arg t|<\phi_0,\ 0\leqq|t|<T_0\}$ and $w\in N\equiv\{w\in X;\|w-A_0^{\alpha}u_0\|<R\}$.

A-3°) For any $t\in\Sigma_0$ and $w\in N$

$$\begin{cases}\text{the resolvent of } A(t,A_0^{-\alpha}w) \text{ contains the left half-plane and}\\ \text{there exists } C_1 \text{ such that } \|(\lambda-A(t,A_0^{-\alpha}w))^{-1}\|\leqq C_1(1+|\lambda|)^{-1}.\end{cases} \tag{2.1}$$

A-4°) The domain $D(A(t,A_0^{-\alpha}w)^h)=D$ of $A(t,A_0^{-\alpha}w)^h$ is independent of $t\in\Sigma_0$ and $w\in N$.

A-5°) The map $\Phi:(t,w)\mapsto A(t,A_0^{-\alpha}w)^hA_0^{-h}$ is analytic from $(\Sigma_0\backslash\{0\})\times N$ to $B(X)$.

A-6°) There exist $C_2,C_3,\sigma, 1-h<\sigma\leqq 1$ such that

$$\|A(t,A_0^{-\alpha}w)^hA(s,A_0^{-\alpha}v)^{-h}\|\leqq C_2 \qquad t,s\in\Sigma_0,\ w,v\in N, \tag{2.2}$$