55. On F^4-Manifolds and Cell-Like Resolutions

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1. Statement of results. For a compact metric space X of dimension n, "a cell-like resolution of X" is defined as a pair (M, f), consisting of an n-dimensional topological manifold M and a cell-like map $f: M \to X$. The purpose of this note is to announce a 4-dimensional version of resolution theorem of generalized manifolds, [1], [2], [5], [6], in terms of F-manifolds whose notion was introduced by Freedman and Quinn [3].

Definition [3]. A topological space M is said to be an F^4-*manifold*, if it is an ENR homology 4-manifold with isolated 1-LC nonmanifold points, disjoint from the boundary. A topological space M is said to be an F^5-*manifold*, if it is an ENR homology 5-manifold whose boundary is collared and is an F^4-manifold, and whose interior is a topological manifold.

The notion of F-manifolds is "a workable substitute" for manifolds in dimension 4. (See [3].)

Main Theorem. *Let X be a 1-connected closed homology 4-manifold, whose nonmanifold set $N(X)$ consists of isolated points. Suppose $X-N(X)$ has a structure of a 1-connected smooth manifold, and $X \times \boldsymbol{R}$ is a 5-dimensional topological manifold. Then there exist a sequence of homology 4-manifolds and cell-like maps between them;*

$$M_0 \xrightarrow{f_0} M_1 \xleftarrow{f_1} M_2 \xrightarrow{f_2} M_3 \xleftarrow{f_3} M_4 = X,$$

where M_i is a homology 4-manifold $(i=1, \cdots, 4)$, M_0 is an F^4-manifold, and f_i is a cell-like map.

The following F^5-version of Quinn's thin h cobordism theorem is essential in proving the main theorem. For the terminology below see [3] and [5].

Theorem A. *Let X be a locally compact metric space, C and D closed subsets of X with $C \supset D$, and ε a positive function on X. Suppose X is locally 1-connected near $C^{2\varepsilon}$, where $C^{2\varepsilon}$ denotes the 2ε-neighborhood of C. Then there exists a positive function $\delta = \delta(X, C, D, \varepsilon)$ on X, having the following property* (*).

(*) *For any compact 1-connected F^5-manifold $(M, \partial_0 M, \partial_1 M)$ with 1-connected boundaries and any proper map $e: M \to X$, satisfying the condition $(1)_\delta$ below, there are an F^5-manifold $(M', \partial_0 M', \partial_1 M')$ and a proper map $e': M' \to X$, satisfying the condition (2).*