50. The Asymptotics of the Potential Functions of One-Sided Stable Processes

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1. Introduction. Let $x(t)$ be a temporally homogeneous independent increments process with only negative jumps, whose cumulant is

$$K(s)=\frac{1}{t}\log \boldsymbol{E}e^{sx(t)}=as+\frac{b}{2}s^2+\int_{-\infty}^{0}\left(e^{sx}-1-\frac{sx}{1+x^2}\right)\Pi(dx),$$

where $s\geqq 0$, $b\geqq 0$ and the Lévy measure Π is a measure which makes the above integral converge. We define $\zeta=\inf\{t\,;x(t)\leqq 0\}$ and $x^0(t)$, $t\in[0,\zeta)$, is the process obtained by killing $x(t)$ at the moment ζ. It is well known that $x^0(t)$ is a Markov process and the resolvent $\boldsymbol{R}_\lambda^0$ of the process $x^0(t)$ is given by

$$\boldsymbol{R}_\lambda^0 f(x)=\boldsymbol{E}_x\int_0^\zeta e^{-\lambda t}f(x(t))dt$$

for $\lambda>0$ and bounded measurable function $f(x)$. Here $\boldsymbol{E}_x$ and $\boldsymbol{P}_x$ are respectively conditional expectation and conditional probability under the condition $x(0)=x$.

In [3] it was proved for $\lambda>0$, $x>0$

$$\boldsymbol{R}_\lambda^0 f(x)=R_\lambda(x)\int_0^\infty e^{-\rho(\lambda)y}f(y)dy-\int_0^x R_\lambda(x-y)f(y)dy,$$

where $\rho(\lambda)$ is a solution of $K(s)=\lambda$, and the Laplace transform of $R_\lambda(x)$ is

$$\int_0^\infty e^{-sx}R_\lambda(x)dx=\frac{1}{K(s)-\lambda}\qquad \text{for } s>\rho(\lambda).$$

We call $R_\lambda(x)$ *resolvent function* and it was shown there exists $R(x)=\lim_{\lambda\to 0}R_\lambda(x)$, which we call *potential function*. If we put $\rho=\lim_{\lambda\to 0}\rho(\lambda)$, it is obvious

$$\int_0^\infty e^{-sx}R(x)dx=\frac{1}{K(s)}\qquad \text{for } s>\rho.$$

As application of this result [2], we obtain the formulas

$$\boldsymbol{E}_x(\zeta)=\frac{R(x)}{\rho}-\int_0^x R(y)dy,$$

$$\boldsymbol{P}_x(x(\zeta)<z,\zeta<\infty)$$
$$=R(x)\int_0^\infty e^{-\rho y}\Pi(-\infty,z-y)dy-\int_0^x R(x-y)\Pi(-\infty,z-y)dy,$$

where $\Pi(-\infty,z-y)=\int_{-\infty}^{z-y}\Pi(du)$.