46. *The First Cohomology Groups of Infinite Dimensional Lie Algebras*

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Introduction. Let L be an infinite dimensional *formal* Lie algebra corresponding to some infinite transformation group. We are interested in the first cohomology group $H^1(L)$ of L with adjoint representation. In this paper we will treat the following two types of infinite dimensional Lie algebras;

(a) infinite dimensional *transitive flat* Lie algebras,

(b) infinite dimensional intransitive Lie algebras $L[W^*]$ whose transitive parts L are infinite and simple.

Throughout this paper, all vector spaces and Lie algebras are assumed to be defined over the field $\boldsymbol{C}$ of complex numbers.

1. Let V be a finite dimensional vector space. We denote by $D(V)$ the Lie algebra of all formal vector fields over V. The Lie algebra $D(V)$ can be written as $D(V)=\prod_{p\geqq 0} V\otimes S^p(V^*)$ (complete direct sum), where $S^p(V^*)$ denotes p-times symmetric tensor of the dual space V^* of V. By *a transitive flat Lie algebra* we mean a Lie subalgebra $L=\prod_{p\geqq -1} \mathfrak{g}_p$ of $D(V)$ satisfying the following conditions:

(1) Each $\mathfrak{g}_p$ is a subspace of $V\otimes S^{p+1}(V^*)$.

(2) $\mathfrak{g}_{-1}=V$ (transitivity condition).

Since L is a Lie algebra, it must hold that

(3) $[\mathfrak{g}_p, \mathfrak{g}_q]\subset \mathfrak{g}_{p+q}$.

A Lie subalgebra $\mathfrak{g}_0$ is called a linear isotropy algebra of L. We say that a Lie algebra $L=\prod_{p\geqq -1} \mathfrak{g}_p$ is *derived* from $\mathfrak{g}_0$ if each $\mathfrak{g}_p$ coincides with the p-th prolongation of $\mathfrak{g}_0$.

We now give two criteria for $H^1(L)$ to be of finite dimension.

Theorem 1. *Let $L=\prod_{p\geqq -1} \mathfrak{g}_p$ be an infinite transitive flat Lie algebra with a semi-simple linear isotropy algebra. Then $H^1(L)$ is finite dimensional.*

Theorem 2. *Let $L=\prod_{p\geqq -1} \mathfrak{g}_p$ be an infinite transitive flat Lie algebra whose linear isotropy algebra $\mathfrak{g}_0$ contains a trivial center. Then $H^1(L)$ is finite dimensional. Furthermore if L is derived from $\mathfrak{g}_0$, then $H^1(L)$ is isomorphic to $\mathfrak{n}(\mathfrak{g}_0)/\mathfrak{g}_0$, where $\mathfrak{n}(\mathfrak{g}_0)$ denotes the normalizer of $\mathfrak{g}_0$ in $\mathfrak{gl}(V)$.*