26. *Lévy's Functional Analysis in Terms of an Infinite Dimensional Brownian Motion. I*

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§ 1. **Introduction.** In his book [1], Paul Lévy has extensively developed a potential theory in an infinite dimensional space.

T. Hida and H. Nomoto have constructed the projective limit $(\mathring{S}_\infty, \mu)$ of the topological stochastic family $\{(\mathring{S}_n, \mu_n)\}$ consisting of the open subsets $\mathring{S}_n$ of the finite dimensional spheres S_n and the restrictions μ_n to $\mathring{S}_n$ of the uniform probability measures on S_n such that $\mu_n(\mathring{S}_n)=1$.

By using this theory, we shall prove the relation:

$$L^2(\mathring{S}_\infty, \mu) = \varprojlim L^2(\mathring{S}_n, \mu_n),$$

and give an interpretation to Lévy's potential theory for Dirichlet problems on the unit ball by introducing the Brownian motion $(\boldsymbol{B}, \boldsymbol{E})$ on an infinite dimensional space $\boldsymbol{E}$ such that $E \supset \mathring{S}_\infty$. We shall also establish the strong Markov property, the uniform continuity of the paths and the skew product formula of the Brownian motion.

§ 2. **Projectively consistent construction of multiple Wiener integrals.** First we reformulate T. Hida and H. Nomoto's results [2] in a slightly different manner from theirs. Let S_n be the sphere with center zero and radius $\sqrt{n+1}$ in the $(n+1)$-dimensional Euclidean space E_{n+1}, and $\mathring{S}_n$ be the open subset of S_n consisting of the points $(x_1, \cdots, x_{n+1})$:

$$\begin{cases} x_1 = \sqrt{n+1} \prod_{i=1}^{n} \sin\theta_i, \\ x_k = \sqrt{n+1} \cos\theta_{k-1} \prod_{i=k}^{n} \sin\theta_i, \qquad (k=2, \cdots, n), \\ x_{n+1} = \sqrt{n+1} \cos\theta_n, \end{cases}$$

with the restriction that $(\theta_1, \cdots, \theta_n) \in \Pi^n$, where $\Pi^n = \{(\theta_1, \cdots, \theta_n);\ 0<\theta_1<2\pi,\ 0<\theta_i<\pi,\ i=2, \cdots, n\}$. We denote by $\pi_{n,m}$ $(n>m)$ the projection of $\mathring{S}_n$ to $\mathring{S}_m$ such that the following is commutative:

$$\begin{array}{ccc} \Pi^n \ni (\theta_1, \cdots, \theta_n) & \longrightarrow & (\theta_1, \cdots, \theta_m) \in \Pi^m \\ \downarrow & & \downarrow \\ \mathring{S}_n & \xrightarrow{\quad \pi_{n,m} \quad} & \mathring{S}_m. \end{array}$$

Set

$$\mathring{S}_\infty = \left\{ x = (x_1, \cdots, x_n, \cdots);\ \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} x_k^2 = 1,\ x_1 \neq 0 \text{ or } x_2 < 0 \right\}$$