25. *Monodromy Preserving Deformation and its Application to Soliton Theory*

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§ 1. **Introduction.** In a preceding article [6], the author investigated the monodromy preserving deformation theory of linear differential equations. The purpose of the present note is to study its relation with the theory of isospectral deformation. In this connection, the reader is referred to the works of Ablowitz *et al.* [1], [2], and to the recent paper of Flaschka-Newell [3] in which they study the link between monodromy and spectrum preserving deformations by a slightly different approach from the present work. Here we show that the soliton theory is naturally incorporated within the framework of the former by considering a degenerate case rather than the "generic" case discussed in [6].

To be specific, the equations dealt with in this paper are the following 2×2 first order systems

(1.1) $PY=0, \qquad P=d/dx-(x^{-2}E+x^{-1}F+G+\sum_{j=1}^{N}H_j/(x-a_j))$

(1.2) $PY=0, \qquad P=d/dx-(xG+F+\sum_{j=1}^{N}H_j/(x-a_j))$

where the eigenvalues of H_j are now assumed to differ by integers. The deformation equations for (1.1)-(1.2) are constructed in a parallel way as in [6]. We give a necessary and sufficient condition for (1.1), (1.2) to be deformed without altering the Stokes multipliers, the global monodromy and the connection matrices, and state that the resulting non-linear equations are completely integrable (Theorems 1, 2), In §4, we sketch the proof of Theorem 1. In § 5, we show that the N-soliton solutions for the sine-Gordon equation are related to the solution of the deformation equations for (1.1).

Further results along the present line will be published elsewhere.

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§ 2. **Construction of the deformation equations for (1.1).** Let U be an open set in C^p. The 2×2 coefficient matrices E, F, G, and H_j $(1\leqq j\leqq N)$ of (1.1) are assumed to be holomorphic in $t=(t_1, \cdots, t_p)$ $\in U$. Note that (1.1) has irregular singularities of rank one at $x=0$, ∞, and regular ones at $x=a_j$ $(1\leqq j\leqq N)$. We make the following as-