89. On Some Series of Regular Irreducible Prehomogeneous Vector Spaces

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Let $\boldsymbol{Q}=\boldsymbol{C}\cdot 1+\boldsymbol{C}\cdot \boldsymbol{e}_1+\boldsymbol{C}\cdot \boldsymbol{e}_2+\boldsymbol{C}\cdot \boldsymbol{e}_1\boldsymbol{e}_2$ be the quaternion algebra over $\boldsymbol{C}$ defined by $\boldsymbol{e}_1^2=\boldsymbol{e}_2^2=-1$ and $\boldsymbol{e}_1\boldsymbol{e}_2=-\boldsymbol{e}_2\boldsymbol{e}_1$. Then the conjugate $\bar{x}$ of an element $x=x_0\cdot 1+x_1\boldsymbol{e}_1+x_2\boldsymbol{e}_2+x_3\boldsymbol{e}_1\boldsymbol{e}_2$ of $\boldsymbol{Q}$ is given by $\bar{x}=x_0\cdot \mathbf{1}-x_1\boldsymbol{e}_1-x_2\boldsymbol{e}_2-x_3\boldsymbol{e}_1\boldsymbol{e}_2$. We define the Cayley algebra (the octanion algebra) $\mathfrak{L}=\boldsymbol{Q}+\boldsymbol{Q}\boldsymbol{e}$ by $(q+r\boldsymbol{e})\cdot(s+t\boldsymbol{e})=(qs-\bar{t}r)+(tq+r\bar{s})\boldsymbol{e}$ for $q,r,s,t\in\boldsymbol{Q}$. Then the conjugate $\bar{y}$ of an element $y=y_1+y_2\boldsymbol{e}$ of $\mathfrak{L}$ is given by $\bar{y}=\bar{y}_1-y_2e$ for $y_1,y_2\in\boldsymbol{Q}$. Put $A_1=\boldsymbol{R}\otimes_{\boldsymbol{R}}\boldsymbol{C}=\boldsymbol{C}\cdot\mathbf{1}$, $A_2=\boldsymbol{C}\otimes_{\boldsymbol{R}}\boldsymbol{C}=\boldsymbol{C}\cdot\mathbf{1}+\boldsymbol{C}\cdot\boldsymbol{e}_1$, $A_4=\boldsymbol{H}\otimes_{\boldsymbol{R}}\boldsymbol{C}=\boldsymbol{Q}$ and $A_8=\mathfrak{L}_{\boldsymbol{R}}\otimes_{\boldsymbol{R}}\boldsymbol{C}=\mathfrak{L}$. Let V_l be the totality of 3×3 hermitian matrices over A_l $(l=1,2,4,8)$ and let $\boldsymbol{G}_l$ be the group $\boldsymbol{SL}(3,A_l)$ $(l=1,2,4)$ and $\boldsymbol{E}_6$ $(l=8)$. Then the group $\boldsymbol{G}_l$ acts on V_l by $\rho_l(g)X=gX{}^t\bar{g}$ for $g\in\boldsymbol{G}$, $X\in V_l$ $(l=1,2,4)$ and $\rho_8=\boldsymbol{\Lambda}_1$. Moreover, for $n\geq 1$, the group $\boldsymbol{G}_l\times \boldsymbol{GL}(n)$ has the action $\rho_l\otimes\boldsymbol{\Lambda}_1$ on $V=V_l\otimes V(n)\cong V_l\oplus\cdots\oplus V_l$ (n-copies) by $X\mapsto(\rho_l(g_1)X_1,\cdots,\rho_l(g_1)X_n)g_2$, for $X=(X_1,\cdots,X_n)\in V$ and $g=(g_1,g_2)\in\boldsymbol{G}_l\times\boldsymbol{GL}(n)$. This triplet $\boldsymbol{P}_{l,n}=(\boldsymbol{G}_l\times\boldsymbol{GL}(n),\ \rho_l\otimes\boldsymbol{\Lambda}_1,\ V_l\otimes V(n))$ is a regular irreducible prehomogeneous vector space for $n=1,2$ and $l=1,2,4,8$. In this article, we give the classification of their orbit spaces, the holonomy diagrams and the b-functions of their relative invariants.

In the case of $l=1$, this work was first done by Prof. M. Sato. In the case of $l=2$, this work was first done in the summar seminor for the study of the prehomogeneous vector spaces in 1974 by the participants including the authors, and reported by J. Sekiguchi in [4].

§1. Any relative invariant $f(X)$ of $\boldsymbol{P}_{l,n}$ $(n=1,2)$ is written as $f(X)=cf_{l,n}(X)^m$ $(c\in\boldsymbol{C}, m\in\boldsymbol{Z})$ with some irreducible polynomial $f_{l,n}(X)$. For an element X of V_l, we can define the determinant $\det X$ (see [1]). Then we have $f_{l,1}(X)=\det X$ for $X\in V_l$. For $n=2$, $f_{l,2}(X)$ is given by the discriminant $(z_1^2z_2^2+18z_0z_1z_2z_3-4z_0z_2^3-4z_1^3z_3-27z_0^2z_3^2)$ of the binary cubic form $\det(uX_1+vX_2)=\sum_{i=0}^3 z_iu^{3-i}v^i$ for $X=(X_1,X_2)\in V_l\oplus V_l$. We have $\deg f_{l,1}=3$ and $\deg f_{l,2}=12$.

§2. Put $\varphi(x)=\begin{pmatrix} x_0+\sqrt{-1}x_1, & -x_2-\sqrt{-1}x_3 \\ x_2-\sqrt{-1}x_3, & x_0-\sqrt{-1}x_1 \end{pmatrix}$ for $x=x_0\cdot 1+x_1\boldsymbol{e}_1+x_2\boldsymbol{e}_2+x_3\boldsymbol{e}_1\boldsymbol{e}_2\in\boldsymbol{Q}$. This gives an isomorphism $\varphi: A_4\rightleftarrows \boldsymbol{M}_2(\boldsymbol{C})$ which induces $A_2\rightleftarrows\left\{\begin{pmatrix} x & 0 \\ 0 & y\end{pmatrix};\ x,y\in\boldsymbol{C}\right\}$ and $A_1\rightleftarrows\left\{\begin{pmatrix} x & 0 \\ 0 & x\end{pmatrix};\ x\in\boldsymbol{C}\right\}$. We define the isomor-

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