81. An Extension of the Aumann-Perles' Variational Problem*)

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1. Introduction. Let $u:[0,1]\times\boldsymbol{R}^l_+\to\boldsymbol{R}$, $x:[0,1]\to\boldsymbol{R}^l_+$ and consider the following problem:

$$\underset{x}{Maximize}\int_0^1 u(t,x(t))dt$$
$$subject\ to$$
$$\int_0^1 x(t)dt=(1,1,\cdots,1).$$

($\boldsymbol{R}^l_+$ designates the non-negative orthant of $\boldsymbol{R}^l$.) The variational problem of this type has a lot of interesting applications to economic analysis (cf. Aumann-Shapley [3], Kawamata [7], and Yaari [9]). Aumann-Perles [2] first examined this problem and established a set of sufficient conditions which assures the existence of an optimal solution. Berliocchi-Lasry [4] and Artstein [1] generalized the problem and proved the existence of solutions respectively in quite different ways.

In this paper, I am going to get a further extension of the problem, the application of which can be seen in recent formulations of welfare economics (cf. Kawamata [7]).

2. An extension of the problem. Let T be a compact metric space, and $\bar{\mu}$ be a non-atomic, positive Radon measure on T with $\bar{\mu}(T)=C<+\infty$. We designate by $\mathfrak{M}_{\bar{\mu}}$ the set of all positive Radon measures μ on T such that

$$\text{(i)}\quad \mu\ll\bar{\mu}\qquad \text{(ii)}\quad \mu(T)\leqq C.$$

Let X be a locally compact Polish space, and let

$$u: T\times X\to\boldsymbol{R}$$
$$g_i: T\times X\to\bar{\boldsymbol{R}}_+\quad;\quad i=1,2,\cdots,l.$$

Then our problem is:

$$\underset{\mu,x}{Maximize}\int_T u(t,x(t))d\mu$$
$$subject\ to$$

$$\text{(I)}\qquad \text{a)}\quad \int_T g_i(t,x(t))d\mu\leqq\omega_i\quad;\quad i=1,2,\cdots,l$$
$$\text{b)}\quad \mu\in\mathfrak{M}_{\bar{\mu}}$$

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