57. *Convergence of Approximate Solutions to Quasi-Linear Evolution Equations in Banach Spaces*

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1. Introduction. In this paper we consider the Cauchy problem for the quasi-linear equation of evolution

(Q) $\quad du(t)/dt+A(t,u(t))u(t)=0, \quad a.e.\ t\in[0,T], \quad u(0)=a,$

under the following assumptions.

(X) X is a Banach space with norm $\|\cdot\|$. There is another Banach space Y, continuously and densely embedded in X. There is an isomorphism S of Y onto X. The norm $\|\cdot\|_Y$ in Y is chosen so that S becomes an isometry.

(I) For each $t\in[0,T_0]$ and $y\in W_R$, $-A(t,y)$ is the infinitesimal generator of a (C_0) semigroup $\{\exp[-sA(t,y)]\}_{s\geqq 0}$ on X such that $\|\exp[-sA(t,y)]\|\leqq e^{\alpha s}$, where T_0, α and R are positive constants and $W_R=\{y\in Y:\|y\|_Y\leqq R\}$.

(II) For each $t\in[0,T_0]$ and $y\in W_R$, there is a bounded linear operator $B(t,y)$ on X into itself such that $SA(t,y)S^{-1}=A(t,y)+B(t,y)$, $\|B(t,y)\|\leqq\lambda$, where λ is a positive number independent of $t\in[0,T_0]$ and $y\in W_R$.

(III) For each $t\in[0,T_0]$ and $y\in W_R$, we have $D(A(t,y))\supset Y$. The restriction of $A(t,y)$ to Y (which is a bounded linear operator on Y into X by the closed graph theorem) satisfies the following:

$$\|A(t,y)-A(t,z)\|_{Y,X}\leqq\mu\|y-z\|, \qquad t\in[0,T_0],\ y,z\in W_R,$$

where μ is a positive constant and $\|\cdot\|_{Y,X}$ is the operator norm in the Banach space of all bounded linear operators on Y into X.

(IV) For each $y\in W_R$ and $x\in Y$, $t\to A(t,y)x$ is continuous in X.

Assumptions (I) and (II) imply that $\exp[-sA(t,y)](Y)\subset Y$ and the restriction of $\exp[-sA(t,y)]$ to Y is a (C_0) semigroup on Y such that $\|\exp[-sA(t,y)]\|_Y\leqq e^{\gamma s}$, where γ is a positive constant. See [1]. Assumption (IV) is somewhat weaker than the corresponding assumption of [1]. It is assumed in [1] that $t\to A(t,y)$ is continuous in $\|\cdot\|_{Y,X}$-norm.

Using the perturbation theory for the linear equation of evolution, Kato [1] studied in detail the Cauchy problem for the quasi-linear equation of evolution. The purpose of this note is to show another approach to (Q). In § 2, we construct approximate solutions to (Q)