56. A Note on the Blowup-Nonblowup Problems in Nonlinear Parabolic Equations

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1. Many studies have been made on the following type of semilinear parabolic equations

$$\frac{\partial}{\partial t}u(x,t)=\Delta u+F(x,t,u), \tag{1.1}$$

especially in connection with the so-called blowing-up problems (cf. [2]). However, hardly any discussion has yet been made on the effect of the coefficient of Δu, as it is a function in u, upon the global behaviour of the solutions. Here, we discuss some subjects in this field. For simplicity, we restrict the spatial dimension to 1, which is not quite essential.

For an interval $I\subset R^1$ and $\alpha\in(0,1)$ we define $H^{2+\alpha}(\infty,I)$ as follows:

$$\begin{cases} H^{2+\alpha}(\infty,I)\equiv\{u(x,t), \text{ defined on } I\times[0,\infty)\text{; for } \forall T\in(0,\infty),\ u, \\ \text{as restricted to } I\times[0,T], \text{ belongs to } H^{2+\alpha}_{T(I)}\}, \end{cases} \tag{1.2}$$

where $H^{2+\alpha}_{T(I)}$ is a Hölder space defined by replacing R^1 by I in the definition of $H^{2+\alpha}_T$ (cf. [3]).

2.1. We consider the following mixed problem for a non-linear parabolic equation:

$$\frac{\partial}{\partial t}u(x,t)=\varphi(u)\frac{\partial^2}{\partial x^2}u+\psi(u),\ (x\in I\equiv[0,a]\ (a>0),\ t\geqq 0), \tag{2.1}$$

$$\begin{cases} u(x,0)=u_0(x)\in H^{2+\alpha}_{(I)}(\geqq 0),\ u(0,t)=u(a,t)=0(t\geqq 0), \\ u_0(0)=u_0(a)=u_0''(0)=u_0''(a)=0, \end{cases} \tag{2.2}$$

where $\varphi(u)$ and $\psi(u)$ are defined on $[0,\infty)$, and are monotonically increasing, non-negative, of the C^1-class on $[0,\infty)$ and of the C^2-class on $(0,\infty)$, and especially $\varphi(0)$ is positive. Without proof we state:

Theorem 2.1 (cf. [7] etc.). *For some $T\in(0,\infty)$, there exists a unique solution $u(x,t)$ for* (2.1)–(2.2) *belonging to $H^{2+\alpha}_{T(I)}$. (Note that $u(x,t)$ is non-negative.)*

We shall state below that, under some conditions on $\varphi(u)$, $\psi(u)$, and u_0, there is a unique solution $u(x,t)$ for (2.1)–(2.2) belonging to $H^{2+\alpha}(\infty,I)$, and that, under some other conditions on them, the solution $u(x,t)$ blows up in a finite time. We remark (cf. [3], [4], etc.) that, in order to show the former, we need only to have *a priori* estimates for $u(x,t)$ such that $|u|^{(0)}_{T(I)}\leqq A(T)(\nearrow(T\nearrow\infty))$, under the assumption that