55. *Parametrix for a Degenerate Parabolic Equation*

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§1. **Introduction.** The purpose of this note is to construct a parametrix of a Cauchy problem 1) for a parabolic equation with a degenerate principal symbol:

$$1)\quad \begin{cases} \left(\dfrac{\partial}{\partial t}+p(x,D)\right)u(x,t)=f(x,t) & \text{on } \boldsymbol{R}^n\times[0,T], \\ u(x,0)=g(x) & \text{on } \boldsymbol{R}^n, \end{cases}$$

$p(x,D)$ being a pseudo-differential operator whose symbol $p(x,\xi)$, independent of t, is in $S^m_{1,0}=L^m_{1/2}$ and has an asymptotic behavior 2);

$$2)\quad p(x,\xi)\sim p_m(x,\xi)+p_{m-1}(x,\xi)+\cdots \text{ as } |\xi|\to\infty,$$

$p_\nu(x,\xi)$ being positively homogeneous in ξ of order ν for $|\xi|\geqq 1$ and the principal symbol p_m being real non-negative ($m>1$).

Melin's result [4] proves the existence of fundamental solution E for 1) under some condition for subprincipal symbols using functional methods. Our method is direct. Under the same condition a complex phase function is given at first in a simple function of a principal symbol and a subprincipal symbol, and amplitudes follow inductively. Consequently a parametrix represented by pseudo-differential operators in $S^0_{1/2\ 1/2}=L^0_0$ is gotten. The parametrix implies the existence of fundamental solution and also Melin's result as a corollary.

§2. **Notations.** Here we employ the Weyl symbol for pseudo-differential operators, that is, a symbol $a(x,\xi)$ defines an operator $a(x,D)$ by 3):

$$3)\quad a(x,D)u(x)=(2\pi)^{-n}\iint e^{i(x-y)\xi}a\left(\frac{x+y}{2},\xi\right)u(y)dyd\xi \text{ for } u\in C_0^\infty.$$

Hence p_{m-1} is the subprincipal symbol in usual sence. $\nabla^k a$ stands for a section of $T^{*k}(T^*\boldsymbol{R}^n)$, k-th symmetric tensor of $T^*(T^*\boldsymbol{R}^n)$, defined by 4) with respect to the canonical coordinate of $T^*\boldsymbol{R}^n$:

$$4)\quad \sum_{|\alpha+\beta|=k} c^k_{\alpha\beta}a^{(\alpha)}_{(\beta)}(d\xi)^\alpha(dx)^\beta;$$

$$c^k_{\alpha\beta}=k!/\alpha!\,\beta! \quad\text{and}\quad a^{(\alpha)}_{(\beta)}=\langle\xi\rangle^{(|\alpha|-|\beta|)/2}\partial^\alpha_\xi\partial^\beta_x a(x,\xi).$$

σ^1 is the canonical two form $d\xi\wedge dx$ on $T^*\boldsymbol{R}^n$. σ^k is its extension onto $T^k(T^*\boldsymbol{R}^n)\times T^k(T^*\boldsymbol{R}^n)$. J_k is the identification map of $T^{*k}(T^*\boldsymbol{R}^n)$ to $T^k(T^*\boldsymbol{R}^n)$ defined by $\sigma^k(u,J_kf)=\langle u,f\rangle$. A bilinear form $\langle J_kf,g\rangle$ on $T^{*k}(T^*\boldsymbol{R}^n)$ is denoted by $\sigma_k(f,g)$. A linear map defined by $\nabla^k a$ from

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