54. *On Unimodal Linear Transformations and Chaos*

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§ 0. **Introduction.** Recently, there appeared many works which investigate how the orbit $\{f_\mu^n(x)\,;\, n\geq 0\}$ starting from an initial point $x\in[0,1]$ behaves asymptotically for a family of continuous maps f_μ from the interval $[0,1]$ into itself with a parameter μ. [1]–[6].

In the present paper we treat the unimodal linear transformations as a simple case of such maps f_μ. In general, we call a continuous map f from $[0,1]$ into itself a unimodal linear transformation if f takes the extremum at c and f is linear on each intervals $[0,c]$ and $[c,1]$, for some $c\in(0,1)$. But we only treat the maps defined by

Definition 0.1. *Let $a>0$, $b>1$ and $a+b-ab\geqq 0$. Let a unimodal linear transformation f_μ with parameter $\mu=(a,b)$ be*

$$(1)\qquad f_\mu(x)=\begin{cases} ax+\dfrac{a+b-ab}{b} & \text{for } 0\leqq x\leqq 1-\dfrac{1}{b}\\[2mm] -b(x-1) & \text{for } 1-\dfrac{1}{b}\leqq x\leqq 1.\end{cases}$$

It is not difficult to see that the general unimodal linear transformations are essentially reduced to f_μ of the Definition 0.1, with some trivial exceptions.

In the present paper we state the results only. The proofs of these results will be given in forthcoming papers "On unimodal linear transformations and chaos. I, II" which will appear in Tokyo Journal. We will treat the case $a=b$ in I and the general case in II in detail.

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§ 1. **Some notations and definitions.** For f_μ defined by (1), let a pair of intervals $\{I_0, I_1\}$, which we will call the fundamental partition of f_μ, be as follows:

Let $I_0=\left[0, 1-\dfrac{1}{b}\right]$ and $I_1=\left(1-\dfrac{1}{b}, 1\right]$ in the case when $f_\mu^n(0)=0$, $f_\mu^i(0)\neq 0$ for $1\leqq i\leqq n-1$ for some natural number n, and the number k defined by

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