53. *Perturbation of Domains and Green Kernels of Heat Equations. III*

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§ 1. Let Ω be a bounded domain in $\mathbf{R}^n$ with smooth boundary γ. Let $\rho(x)$ be a smooth function on γ and ν_x be the exterior unit normal vector at $x\in\gamma$. For sufficiently small $\varepsilon\geq 0$, let Ω_ε be the bounded domain whose boundary γ_ε is defined by

$$\gamma_\varepsilon=\{x+\varepsilon\rho(x)\nu_x\,;\,x\in\gamma\}.$$

Let $G_\varepsilon(x,y)$ be the Green's function of the Dirichlet boundary value problem of the Laplacian on Ω_ε. We abbreviate $G_0(x,y)$ as $G(x,y)$. Put

$$\delta^k G(x,y)=\frac{\partial^k}{\partial\varepsilon^k}G_\varepsilon(x,y)\big|_{\varepsilon=0}\qquad\text{for } k=1,2.$$

Put

$$\nabla_z a(z)\cdot\nabla_z b(z)=\sum_{j=1}^n\frac{\partial a}{\partial z_j}(z)\frac{\partial b}{\partial z_j}(z)\qquad\text{for any } a(z),\,b(z)\in C^\infty(\Omega).$$

By $H_1(z)$ we denote the first mean curvature of γ at z. Then, Garabedian-Schiffer [1] proved the following:

$$\begin{aligned}\delta^2G(x,y)=&-\int_\gamma\frac{\partial G(x,z)}{\partial\nu_z}\frac{\partial G(y,z)}{\partial\nu_z}(n-1)H_1(z)\rho(z)^2d\sigma_z\\ &+2\int_\Omega\nabla_z\delta^1G(x,z)\cdot\nabla_z\delta^1G(y,z)dz.\end{aligned}\tag{1.1}$$

Here $\partial/\partial\nu_z$ denotes the exterior normal derivative with respect to z and $d\sigma_z$ denotes the surface element of γ.

Let $U_\varepsilon(x,y,t)$ denote the fundamental solution of the heat equation with the Dirichlet boundary condition on γ_ε. Put

$$\delta^k U(x,y,t)=\frac{\partial^k}{\partial\varepsilon^k}U_\varepsilon(x,y,t)\big|_{\varepsilon=0}$$

for $k=1,2$. We abbreviate $\delta^1U(x,y,t)$ as $\delta U(x,y,t)$. In [2] and [3] the author gave explicit representation of $\delta U(x,y,t)$, that is

$$\delta U(x,y,t)=\int_0^t d\tau\int_\gamma\frac{\partial U(x,z,t-\tau)}{\partial\nu_z}\frac{\partial U(y,z,\tau)}{\partial\nu_z}\rho(z)d\sigma_z.\tag{1.2}$$

We can prove the following

Theorem 1. *For $x,y\in\Omega$, $t>0$*

$$\begin{aligned}&\delta^2U(x,y,t)\\ &=-\int_0^t d\tau\int_\gamma\frac{\partial U(x,z,t-\tau)}{\partial\nu_z}\frac{\partial U(y,z,\tau)}{\partial\nu_z}(n-1)H_1(z)\rho(z)^2d\sigma_z\end{aligned}\tag{1.3}$$