46. *Global Branching Theorem for Spatial Patterns of Reaction-Diffusion System*

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Introduction. In this note, we show the main idea to prove the existence of the global branch for spatial patterns of reaction-diffusion system as follows

$$0=\beta^2 u_{xx}+f(u,v), \qquad x\in I \parallel (0,1) \tag{1.1}$$

$$0=\frac{1}{\alpha}v_{xx}+g(u,v) \tag{1.2}$$

$$u_x(0)=u_x(1)=v_x(0)=v_x(1)=0, \tag{1.3}$$

where $1/\alpha$ and β^2 are diffusion coefficients. Our principal assumptions are

(A-1) (1) has a invariant rectangle R (see [1]).

(A-2) (1) has a unique positive constant solution $(\bar{u},\bar{v})\in R$ such that Jacobi matrix of F at $(\bar{u},\bar{v})$ is stable, i.e., real parts of its eigenvalues are negative, where $F={}^t(f(u,v),g(u,v))$.

(A-3) 0-level curve of f is sigmoidal and that of g intersects it transversally (see [2, Figs. 1–3]).

(A-4) α is fixed to be sufficiently small.

This type of system satisfying (A-1)–(A-4) appears in many fields such as population dynamics, morphogenesis and so on (see [3], [4]). Under these assumptions, we can obtain the global result Theorem 3 for the bifurcating branch from $(\bar{u},\bar{v})$ when β varies as a bifurcation parameter. We note that Theorem 3 says not only the global existence of the bifurcating branch but also its asymptotic behavior when β tends to zero.

Remark 1. We use the word "exist globally" in the sense that the bifurcating branch in $\boldsymbol{R}^+\times(H_N^2(I))^{2}$ *) can be extendible for any small β, i.e., the β-section of it for any small β is not empty and contains non-trivial solutions.

For the local bifurcation problem of (1), many works have been done and for the details, see, for instance, [5]. When β leaves the critical point, we have, in general, little informations about how the bifurcating branch changes, however, if the above four assumptions are satisfied, we can describe the global picture of the bifurcating

*) $\boldsymbol{R}^+=(0,+\infty)$. $H_N^2(I)$=closure of $\{\cos(n\pi x)\}_{n=0}^{\infty}$ in the usual Sobolev space $H^2(I)$.