36. Studies on Holonomic Quantum Fields. XIV

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The present article is a direct continuation of our preceding note [1], where deformation theory was discussed in connection with the Riemann-Hilbert problem for Euclidean Dirac equations. We are particularly interested in the step function limit of the matrix $M(\xi)$; in other words the Green's function $w(x, x')$ is now required to be multi-valued, having a monodromic property $w(x, x') \mapsto e^{2\pi i L_\nu} w(x, x')$ when continued around 2-codimensional submanifolds ("Bags") $B_\nu = \{f_\nu = 0, \bar{f}_\nu = 0\}$. Formally the variational formula XIII-(7) [1] then takes the form

$$(1) \qquad \frac{1}{2\pi i}\delta w(x, x') = \sum_\nu \int_{X^{\mathrm{Euc}}} d^s y \cdot w(x, y) \Delta_\nu(y) L_\nu w(y, x')$$

$$\Delta_\nu(y) = \frac{1}{2i}(\partial f_\nu(y) \delta \bar{f}_\nu(y) - \delta f_\nu(y) \partial \bar{f}_\nu(y)) \delta(f_{\nu 1}(y)) \delta(f_{\nu 2}(y))$$

with $f_\nu(y) = f_{\nu 1}(y) + i f_{\nu 2}(y)$. However the meaning of (1) needs to be made precise, since $w(x, x')$ has a regular singularity along B_ν. In this note we perform this procedure in the 2-dimensional (massless and massive) case, and show that the resulting equations are exactly those obtained previously ((2.3.38) in [2] and (3.3.53) in [3]).

We use the following convention:

$$\gamma^1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \ \gamma^2 = \begin{pmatrix} & -i \\ i & \end{pmatrix}, \ \partial\!\!\!/ = \begin{pmatrix} & \partial \\ \bar{\partial} & \end{pmatrix}, \ \partial = \partial_1 - i\partial_2, \ \bar{\partial} = \partial_1 + i\partial_2.$$

1. The Riemann-Hilbert problem for the Euclidean Dirac equation in the sense of [1] has a special feature when the space dimension is 2 and the mass vanishes. Let us restate the problem in this case. As in [1] we denote by D^+ a bounded domain in $X^{\mathrm{Euc}} = \boldsymbol{R}^2$, and let $D^- = X^{\mathrm{Euc}} - \bar{D}^+$, $\partial D^+ = \Gamma$. We set $z = (x^1 + ix^2)/2$, $\bar{z} = (x^1 - ix^2)/2$. Given a real analytic $N \times N$ matrix M on Γ, we are to find a $2N \times 2N$ matrix

$$w = \begin{pmatrix} w_1 & w_2 \\ w_3 & w_4 \end{pmatrix}$$

such that

$$(2) \quad (\mathrm{i}) \quad -\begin{pmatrix} & \partial \\ \bar{\partial} & \end{pmatrix} w(z, \bar{z}; z', \bar{z}') = \delta(x^1 - x'^1)\delta(x^2 - x'^2) \quad (x, x' \notin \Gamma)$$

$$(\mathrm{ii}) \quad |w(z, \bar{z}; z', \bar{z}')| = O\left(\frac{1}{|z|}\right) \quad (|z| \to \infty)$$

$$(\mathrm{iii}) \quad w(\zeta^+, \bar{\zeta}^+; z', \bar{z}') = M(\zeta, \bar{\zeta}) w(\zeta^-, \bar{\zeta}^-; z', \bar{z}') \quad (\zeta, \bar{\zeta}) \in \Gamma$$