34. *Family of Varieties Dominated by a Variety*

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§1. **Introduction.** Let k be an algebraically closed field of characteristic zero. We assume all varieties are defined over k. In this note we prove a finiteness theorem for the isomorphism classes of canonically polarized varieties dominated by any given variety.

It was proved by Severi that only finitely many (up to isomorphism) curves of genus $\geqq 2$ can be dominated by a given variety over any algebraically closed field (cf. [16]).

As a generalization of this result in higher dimensional cases over k, we shall prove the following

Main Theorem. *Only finitely many (up to isomorphism) canonically polarized varieties can be dominated by a given variety.*

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§2. **Statement of results.** We fix our notation. In this note, by a "variety" we shall always mean a proper integral algebraic space over k. A non-singular variety will be said to be canonically polarized if the canonical invertible sheaf is ample.

We denote by Θ_X, Ω_X^n and K_X the sheaf of regular tangent vectors, the sheaf of differential n-forms and the canonical invertible sheaf on a non-singular variety X, respectively.

Let X be a variety. $\mathcal{F}(X)$ denotes the set $\{(f, Y)\}$ of pairs (f, Y) each of which consists of a projective non-singular variety Y and a surjective morphism $f: X \to Y$. $(f_1, Y_1), (f_2, Y_2) \in \mathcal{F}(X)$ are said to be isomorphic to each other (or isomorphic in the strong sense) if there is an isomorphism $g: Y_1 \to Y_2$ (or $g \circ f_1 = f_2$).

$\mathcal{F}^a(X)$ denotes the subset of $\mathcal{F}(X)$ consisting of $\{(f, Y)\}$ with canonically polarized varieties $\{Y\}$.

Now our Main Theorem can be stated as follows:

Theorem 1. *$\mathcal{F}^a(X)$ is finite up to isomorphism in the strong sense.*

We can also show the following

Theorem 2. *$\mathcal{F}(X)$ is at most countable up to isomorphism.*

Shortly speaking at most countable (up to isomorphism) non-singular projective varieties can be dominated by X.