32. *Micro-Local Cauchy Problems and Local Boundary Value Problems*

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In these notes we present existence theorems of micro-local Cauchy problems for pseudo-differential operators of Fuchsian type and of local boundary value problems for a class of linear partial differential operators. These theorems are proved by applying the following; first, the Cauchy-Kovalevskaja theorem in the sense of Bony-Schapira [2] for pseudo-differential operators (of Fuchsian type), which we mention in § 1; and secondly, the method of analytic continuation developed in Kashiwara-Kawai [4].

§ 1. The Cauchy-Kovalevskaja theorem for pseudo-differential operators of Fuchsian type. Let $(t, z)=(t, z_1, \cdots, z_n)\in X=\boldsymbol{C}\times\boldsymbol{C}^n$. We use the notation $D_t=\partial/\partial t$ and $D_z^\alpha=(\partial/\partial z_1)^{\alpha_1}\cdots(\partial/\partial z_n)^{\alpha_n}$ for $\alpha=(\alpha_1, \cdots, \alpha_n)$. Let

$$P=t^kD_t^m+A_1(t, z, D_z)t^{k-1}D_t^{m-1}+\cdots+A_k(t, z, D_z)D_t^{m-k}$$
$$+\cdots+A_m(t, z, D_z)$$

be a pseudo-differential operator of finite order in the sense of Sato-Kawai-Kashiwara [6] which is defined on an open subset of the cotangential projective bundle P^*X of X. We assume the following conditions:

(A.1) $0\leqq k\leqq m$;

(A.2) $\operatorname{ord} A_j(t, z, D_z)\leqq j$ for $j=1, \cdots, m$;

(A.3) $\operatorname{ord} A_j(0, z, D_z)\leqq 0$ for $j=1, \cdots, k$.

Then P is said to be of *Fuchsian type with weight* $m-k$ (cf. Baouendi-Goulaouic [1] and Tahara [7]). We set

$$a_j(z, \zeta)=\sigma_0(A_j(0))(z, \zeta) \qquad \text{for } j=1, \cdots, k,$$

where σ_0 denotes the principal symbol of order 0, and $(z, \zeta\infty)$ is a point of $P^*\boldsymbol{C}^n$. The *indicial equation* associated with P is defined by

$$\lambda(\lambda-1)\cdots(\lambda-m+1)+\lambda(\lambda-1)\cdots(\lambda-m+2)a_1(z, \zeta)$$
$$+\cdots+\lambda(\lambda-1)\cdots(\lambda-m+k+1)a_k(z, \zeta)=0,$$

and its roots are called the *characteristic exponents* of P, which we denote by

$$\lambda=0, \cdots, m-k-1, \lambda_1(z, \zeta), \cdots, \lambda_k(z, \zeta).$$

For the sake of simplicity, we assume that $A_j(t, z, D_z)$ is defined on a neighborhood of ϖ for $j=1, \cdots, m$, where