29. *The Spectrum of the Laplacian and Smooth Deformation of the Riemannian Metric*

By Masao TANIKAWA
Department of Mathematics, University of Tokyo

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§ 1. **Introduction.** Let M be an n-dimensional compact connected C^∞ manifold (with or without boundary ∂M). Every Riemannian metric g of M determines the Laplace-Beltrami operator Δ_g. We consider the eigenvalue problem for $-\Delta_g$ (under Dirichlet condition);

$$\text{(1.1)} \qquad \begin{cases} (-\Delta_g-\lambda)u(x)=0 & \\ u(x)=0 & (\text{if } \partial M \neq \phi). \end{cases}$$

Let $0 \leq \lambda_0 \leq \lambda_1 \leq \lambda_2 \cdots$ be the eigenvalues of the problem (1.1). These are determined by the metric g. The totality of Riemannian metrics of class C^∞ which differ from a fixed metric g_0 only on an open set $U \subset M$ forms a separable Fréchet manifold B.

Theorem A. *If* $\dim M = n \geq 2$, *then there exists a residual subset* $\Gamma \subset B$ *such that all eigenspaces of* $-\Delta_g$ *are one dimensional for any* $g \in \Gamma$.

We call a subset Γ residual if it is a countable intersection of open dense subsets. Since a residual set is dense and a second category by virtue of Baire's theorem, Theorem A implies that for almost all $g \in B$ the eigenvalues of problem (1.1) are all simple.

In our proof we follow the idea of Uhlenbeck [6], who has already obtained the similar result in the case that those metrics are of class C^k $(n+3 \leq k < +\infty)$. But the first transversality theorem of her can not be applied to our case, since B is not a Banach manifold. We need the following Fréchet manifold version of the transversality theorem.

Theorem B. *Let* E, F *and* G *be strong ILH manifolds of class* C^r. *Assume that* E *and* F *are separable. Let the mapping* $f: E \times F \to G$ *be a* C^r*-strong ILH mapping satisfying the following conditions;*

(a) *For every* $u \in E \times F$, *every* $k \in N(d)$,

$$\text{(1.2)} \qquad \|(Df^k)_u \delta u\|_k \geq C_u \|\delta u\|_k - D_u^k \|\delta u\|_{k-1},$$

where $\delta u \in T_u(E \times F)$, C_u *and* D_u^k *are positive constants and* C_u *is independent of* k.

(b) *There exists* $p \in G$ *such that* p *is a regular value of* f. *(That is for any* $u \in f^{-1}(p)$ *the Fréchet derivative* $(Df)_u$ *is onto.)*

(c) *For every* $b \in F$, $f_b = f(\ ,b): E \to G$ *is a strong ILH Fredholm mapping with index* $< r$.

Then the set $\{b \in F\,;\, p$ *is a regular value of* $f_b\}$ *is residual in* F.