28. The Eigenvalues of the Laplacian and Perturbation of Boundary Condition

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§. **Introduction.** Let Ω be a bounded domain in $\mathbf{R}^n$ with C^∞ boundary γ. Consider eigenvalue problem for the Laplacian with the third boundary condition;

$$(1) \qquad (-\Delta-\lambda)u(x)=0, \qquad x\in\Omega,$$

$$(2)_\rho \qquad \frac{\partial u}{\partial \nu_x}(x)+\rho(x)u(x)=0, \qquad x\in\gamma,$$

where ν_x denotes the exterior unit normal vector at x, and $\rho(x)$ is a function in the Hölder space $C^{2+\theta}(\gamma)$ $(0<\theta<1)$.

Let $\lambda_1\leq\lambda_2\leq\lambda_3\leq\cdots$ be the eigenvalues of the problem (1), $(2)_\rho$, then these are functionals of ρ.

Our main result is

Theorem 1. *There is a residual subset B of the Banach space $C^{2+\theta}(\gamma)$ such that for any $\rho\in B$ all the eigenspaces of the problem* (1), $(2)_\rho$ *are of dimension one.*

We call a subset B in $C^{2+\theta}(\gamma)$ residual if it is a countable intersection of open dense subsets of $C^{2+\theta}(\gamma)$.

In Fujiwara-Tanikawa-Yukita [2], they studied the eigenvalue problem of the Laplacian with Dirichlet condition at the boundary γ. Their result is as follows;

(A) If the boundary γ of domain is in residual subset of the Hilbert manifold of the totality of the boundary, then all eigenvalues are simple eigenvalue.

Their proof heavily depends on the abstract transversality theorem of Banach manifold given by Uhlenbeck [3]. Also they used Hadamard's variational formula and showed that the theorem of Uhlenbeck is applicable to their proof of (A).

In our case too, we shall use a variational formula of $(-\Delta+M)^{-1}$, M being large, under the perturbation of $\rho(x)$. This will be proved in Theorem 2.

§1. **Variational formula of the Green kernel under the perturbation of boundary condition.** Let m be a fixed number satisfying $m<\lambda_1$, where λ_1 is the smallest eigenvalue of $-\Delta$ with the boundary condition $(2)_\rho$. Let $G_\rho(x,y)$ be the Green kernel of $-\Delta-m$ with the condition $(2)_\rho$. We fix $\kappa(x)\in C^{2+\theta}(\gamma)$. Then we have the following