59. *Absolute Continuity of Probability Laws of Wiener Functionals*

By Ichiro SHIGEKAWA
Department of Mathematics, Kyoto University
(Communicated by Kôsaku YOSIDA, M. J. A., Oct. 12, 1978)

1. The *Wiener space*, which is a typical example of abstract Wiener spaces introduced by L. Gross [1], is a triple (B, H, μ) where B is a Banach space consisting of all real valued continuous functions $x(t)$ $(x(0)=0)$ defined on the interval $[0,1]$ with norm $\|x\|=\sup_{0\leqslant t\leqslant 1}|x(t)|$, H is a Hilbert space consisting of absolutely continuous functions $x(t)$ $(x(0)=0)$ such that $x'(t)\in L^2[0,1]$ with inner product

$$\langle x, y\rangle_H=\int_0^1 x'(t)y'(t)dt$$

and μ is the Wiener measure, i.e., the Borel probability measure on B such that

$$\int_B e^{i(h,x)}\mu(dx)=\exp\left\{-\frac{1}{2}\langle h, h\rangle_H\right\}, \tag{1}$$

where $h\in B^*\subset H$ and $(\ ,\ )$ is a natural paring of B^* and B. It is readily seen that $\{x(t)\,;\,0\leqslant t\leqslant 1\}$ is a standard Wiener process on the probability space (B,μ). A real-valued (or more generally, a Banach space-valued) measurable function defined on the probability space (B,μ) is called a *Wiener functional.* Two Wiener functionals $F_1(x)$ and $F_2(x)$ are identified if $\mu\{x\,;\,F_1(x)\neq F_2(x)\}=0$. Typical examples of Wiener functionals are solutions of stochastic differential equations or multiple Wiener integrals (see Itô [2]).

Malliavin [3] introduced a notion of derivatives of Wiener functionals and applied it to the absolute continuity of the probability law induced by a solution of stochastic differential equations at a fixed time. Here, we define the derivatives of Wiener functionals in a somewhat different way and rephrase a theorem of Malliavin. We will apply it to the absolute continuity of the probability law induced by a system of multiple Wiener integrals.

2. Let (B,H,μ) be the Wiener space or more generally, any abstract Wiener space. Let E be a Banach space, F be a mapping from B into E, and $\mathcal{L}(B,E)$ denote the space of all bounded linear operators from B into E. If there exists an operator $T\in\mathcal{L}(B,E)$ such that

$$F(x+y)-F(x)=T(y)+o(\|y\|) \qquad \text{as } \|y\|\to 0\ (y\in B), \tag{2}$$

then F is said to be *B-differentiable at* $x\in B$, and the operator T is called the *B-derivative* (or Fréchet derivative) *of* F *at* $x\in B$, $F'(x)$ in