35. Classification Theory of Non-Complete Algebraic Surfaces

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In this paper we shall show that almost all theorems in classification theory of algebraic surfaces by Enriques, Kodaira, Iitaka, Mumford, Bombieri, etc. can be extended to the case of non-complete algebraic surfaces. We use the following notation:

X: a non-singular algebraic surface (this is the object of the study).

$\bar{X}$: a non-singular complete algebraic surface which contains X as a Zariski open subset.

$D=\bar{X}-X$: the complement of X in $\bar{X}$. We assume that D has only normal crossings.

$\kappa(\bar{X})$ (resp. $\bar{\kappa}(X)$): the Kodaira (resp. logarithmic Kodaira) dimension of $\bar{X}$ (resp. X).

$P_m(\bar{X})$ (resp. $\bar{P}_m(X)$): the m-genus (resp. logarithmic m-genus) of $\bar{X}$ (resp. X) (for the definitions, see [4]).

K: the canonical sheaf of $\bar{X}$.

[ ]: the integral part.

For the sake of simplicity, we shall work only on the ground field $\boldsymbol{C}$, that is, in the case of characteristic zero.

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1. We first construct a relatively minimal model (or a super model) of $(X, \bar{X}, D)$.

Theorem 1. *If $\bar{\kappa}(X)\geqslant 0$, then there exist a non-singular complete surface $\bar{X}_m$, a divisor D_m with coefficients in $\boldsymbol{Q}$ on $\bar{X}_m$ and a birational morphism $f:\bar{X}\to\bar{X}_m$ satisfying the following conditions:*

(1) *$D_m=\sum_i d_iD_i$, $0<d_i\leqslant 1$, where the D_i are irreducible divisors on $\bar{X}_m$.*

(2) *$f^*(K_m+D_m)$ is the arithmetically effective component of $K+D$ in the sense of Zariski* (see Definition 7.6 and Theorem 7.7 of [10]), *where K_m is the canonical sheaf of $\bar{X}_m$.*

$(\bar{X}_m, D_m)$ is obtained by a succession of two kind of steps from $(\bar{X}, D)$ as follows: we denote an intermediate stage by $(\bar{X}', D')$, where $\bar{X}'$ is a non-singular complete algebraic surface and D' is a divisor with