33. G-Manifolds and G-Vector Fields with Isolated Zeros

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(Communicated by Kôsaku YOSIDA, M. J. A., May 12, 1978)

Let G be a finite group. A G-manifold is a smooth manifold M together with a smooth G-action on M, and a (continuous) G-vector field on a G-manifold M is a continuous G-equivariant cross section of the tangent bundle $\tau(M)$ of M. The object of this paper is to apply the equivariant homotopy theory of representation spheres [4] to remove isolated zeros of G-vector fields.

1. Preliminaries. Let M be a G-manifold. For any $x \in M$, G_x denotes the isotropy subgroup at x. For any subgroup H of G, define $M_H=\{x \in M \mid G_x=H\}$ and $M^H=\{x \in M \mid H \subset G_x\}$. Then M_H and M^H are submanifolds of M. Let $s: M \to \tau(M)$ be a G-vector field on M. s induces a vector field $s^H: M^H \to \tau(M^H)$ on M^H by restricting s on M^H.

Recall the index of a vector field s on M at an isolated zero $z \in M - \partial M$. The index is denoted by $\operatorname{ind}(z; s)$, and defined to be the degree of the map

$$f=\frac{d\varphi \circ s \circ \varphi^{-1}}{\|d\varphi \circ s \circ \varphi^{-1}\|}: S^{n-1} \to S^{n-1},$$

where φ is a chart from a small neighborhood of z into $\boldsymbol{R}^n$ taking z to 0, and $n=\dim M$. The map f describes the behavior of s near z. When M is a G-manifold and s is a G-vector field, we may take φ so as to be a G_z-equivariant chart from a G_z-invariant neighborhood of z into an orthogonal representation V of G_z taking z to 0. Moreover, the map f is a G_z-equivariant map from $S(V)$ to itself, where $S(V)$ is the unit sphere in V. For any subgroup H of G_z, z is also an isolated zero of s^H, and we see $\operatorname{ind}(z; s^H)=\deg f^H$, where $f^H: S(V)^H \to S(V)^H$ is the restriction of f on $S(V)^H$.

Convention. For the only map $f: \phi \to \phi$ of an empty set, define $\deg f=1$. So the index of a vector field on a 0-dim manifold at each point is 1. For a map $f: S^0 \to S^0$, define $\deg f=1$ if f is the identity, $\deg f=0$ if f maps S^0 to one point, and $\deg f=-1$ if f interchanges the two points of S^0.

2. Removing zeros. Theorem 1. *Let G be a finite abelian group, and K a subgroup of G. Let s be a G-vector field on a G-manifold M. Let A be a connected component of M_K, and $\{z_1, z_2, \cdots, z_p\}$ the zeros of s on A. Assume that all z_i's are isolated zeros of s and are off ∂M, and assume that for any subgroup H of K,*