31. *On the Absolute Nörlund Summability of Orthogonal Series*

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§1. Let $\sum a_n$ be any given infinite series with s_n as its n-th partial sum. If $\{p_n\}$ is a sequence of constants, real or complex, and

$$P_n = p_0 + p_1 + \cdots + p_n\,; P_{-k} = p_{-k} = 0, \qquad \text{for } k \geqq 1,$$

then the Nörlund mean t_n of $\sum a_n$ is defined by

$$t_n = \frac{1}{P_n} \sum_{k=0}^{n} p_{n-k} s_k = \frac{1}{P_n} \sum_{k=0}^{n} P_{n-k} a_k, \qquad (P_n \neq 0). \tag{1.1}$$

If the series

$$\sum_{n=1}^{\infty} |t_n - t_{n-1}| \tag{1.2}$$

converges, then the series $\sum a_n$ is said to be absolutely summable (N, p_n), or summable $|N, p_n|$.

In the special cases in which $p_n = A_n^{\alpha-1} = \binom{n+\alpha-1}{\alpha-1}$, $\alpha > 0$ and $p_n = 1/(n+1)$, summability $|N, p_n|$ are the same as the summability $|C, \alpha|$ and the absolute harmonic summability, respectively.

Let $\{\varphi_n(x)\}$ be an orthonormal system defined in the interval (a, b). We suppose that $f(x)$ belongs to $L^2(a, b)$ and

$$f(x) \sim \sum_{n=0}^{\infty} a_n \varphi_n(x).$$

By $E_n^{(2)}(f)$, we denote the best approximation to $f(x)$ in the metric of L^2 by means of polynomials $\sum_{k=0}^{n-1} a_k \varphi_k(x)$, i.e., $\{E_n^{(2)}(f)\}^2 = \sum_{k=n}^{\infty} |a_k|^2$. We write

$$W_k = \frac{1}{k} \sum_{n=k}^{\infty} \frac{n^2 p_n^2 p_{n-k}^2}{P_n^4} \left(\frac{P_n}{p_n} - \frac{P_{n-k}}{p_{n-k}} \right)^2 \tag{1.3}$$

and

$$\Delta \lambda_n = \lambda_n - \lambda_{n-1}.$$

A denotes a positive absolute constant that is not always the same.

§2. The purpose of this paper is to give a general theorem on the almost everywhere summability $|N, p_n|$ of orthogonal series and deduce several known and new results from the theorem by the similar method as that used by Ul'yanov [7].

Our theorem reads as follows:

Theorem 1. *Let $\{\Omega(n)\}$ be a positive sequence such that $\{\Omega(n)/n\}$*