22. *On the Acyclicity of Free Cobar Constructions. II*

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Examle 1. Let M be a connected simplicial complex and x_0 be a base point of M. Let N be a maximal tree in M containing x_0. We denote by $C_*(M/N)=\oplus_{p=0}^{\infty} C_p(M/N)$ the CW-complex with the only one vertex x_0. Let G be the edge path group of M/N with the base point x_0. The set of all reduced closed pathes in M/N with the base point x_0 forms a free group F. Let L be the two-sided ideal of $\boldsymbol{Z}[F]$, generated by the elements of the form

$$\delta_1 T\langle v_0, v_1, v_2\rangle = \widetilde{T\langle v_0, v_1\rangle}\cdot\widetilde{T\langle v_1, v_2\rangle}-\widetilde{T\langle v_0, v_2\rangle} \tag{1.1}$$

for any reduced 2-simplex T of $M/N: \langle v_0, v_1, v_2\rangle \to M/N$, where $\widetilde{T\langle v_i, v_j\rangle} \in F$. We define X^f as $\boldsymbol{Z}[F]\otimes C_*(M/N)$. For a reduced n-simplex T regarded as an element of X_n^f or A_{n-1} the formulae for ∂_n^f or δ_{n-1} are given as follows respectively:

$$\begin{aligned}&\partial_n^f T\langle v_0, v_1, \cdots, v_n\rangle = \widetilde{T\langle v_0, v_1\rangle}\cdot T\langle v_1, \cdots, v_n\rangle\\ &\quad+\textstyle\sum_{i=1}^{n}(-1)^i\cdot T\langle v_0, v_1, \cdots, v_{i-1}, v_{i+1}, \cdots, v_n\rangle, \quad n\geqq 2 \quad\text{and}\\ &\partial_1^f T\langle v_0, v_1\rangle = \widetilde{T\langle v_0, v_1\rangle}-1 \in \boldsymbol{Z}[F]^+, \quad n=1,\end{aligned} \tag{1.2}$$

where $\widetilde{T\langle v_0, v_1\rangle}$ lies in F, and

$$\begin{aligned}&\delta_{n-1} T\langle v_0, v_1, \cdots, v_n\rangle = \widetilde{T\langle v_0, v_1\rangle}\cdot T\langle v_1, \cdots, v_n\rangle\\ &\quad+\textstyle\sum_{i=1}^{n-1}(-1)^i\cdot T\langle v_0, v_1, \cdots, v_{i-1}, v_{i+1}, \cdots, v_n\rangle\\ &\quad+(-1)^n\cdot T\langle v_0, v_1, \cdots, v_{n-1}\rangle\cdot\widetilde{T\langle v_{n-1}, v_n\rangle}\\ &\quad+\textstyle\sum_{i=2}^{n-2}(-1)^{i-1}\cdot T\langle v_0, v_1, \cdots, v_i\rangle\cdot T\langle v_i, \cdots, v_n\rangle, \quad n\geqq 3,\end{aligned} \tag{1.3}$$

where $\widetilde{T\langle v_0, v_1\rangle}$ and $\widetilde{T\langle v_{n-1}, v_n\rangle}$, $n\geqq 3$, $\widetilde{T\langle v_0, v_1\rangle}$, $\widetilde{T\langle v_1, v_2\rangle}$ and $\widetilde{T\langle v_0, v_2\rangle}$, $n=2$ are in F. The free cobar construction (A, δ) thus defined satisfies Assumptions 1 and 2 in [3]. This is nothing but a modification of Adams cobar construction. So by [3]

Theorem 1. *$\mathrm{H}_p(A)\cong(0)$, $p\geqq 1$ and $\mathrm{H}_0(A)\cong \boldsymbol{Z}[G]$ if and only if M is a $\mathrm{K}(\Pi, 1)$ space.*

This is also related with Pfeiffer-Smith-Whitehead identity relations [4].

Example 2. Let $\mathfrak{G}$ be a Lie algebra over $\boldsymbol{Z}$ and $\mathcal{E}(\mathfrak{G})$ or $T(\mathfrak{G})$ be its envelopping algebra or tensor algebra respectively. We consider the normalized standard complex (X, ∂) on $\mathcal{E}(\mathfrak{G})$, $X=\mathcal{E}(\mathfrak{G})\otimes\Lambda^*\mathfrak{G}$, where $\Lambda^*(\mathfrak{G})$ denotes the exterior algebra of $\mathfrak{G}$. We put $X^f=T(\mathfrak{G})\otimes\Lambda^*\mathfrak{G}$ and define for a $x_1\wedge x_2\wedge\cdots\wedge x_n \in X_n^f$, $x_j\in\mathfrak{G}$, $1\leqq j\leqq n$,