20. On Multivalent Functions in Multiply Connected Domains. II

By Hitoshi ABE
Department of Applied Mathematics, Faculty of Engineering, Ehime University

(Communicated by Kôsaku YOSIDA, M. J. A., May 12, 1977)

1. Introduction. In the preceding paper [1] we extended Rengel's results ([4] or [3]) to the case of circumferentially mean p-valent functions. In this paper we shall treat the case of areally mean p-valent functions defined as follows.

Let $n(R,\Phi)$ denote the number of roots of the equation $f(z)=w=\mathrm{Re}^{i\Phi}$ in a domain D. If for a certain positive integer p,

$$\int_0^R \left(\int_0^{2\pi} n(R,\Phi)d\Phi\right) RdR \leq p\pi R^2 \qquad (0\leq R<\infty), \tag{1.1}$$

then $f(z)$ is called to be areally mean p-valent (cf. [2]).

As defined in [1], D_1, D_2, D_3, D_4, D_5 and D_6 denote the n-ply connected, representative domains of the following types respectively.

D_1: an annulus, $(0<)\ r_1<|z|<r_2\ (<\infty)$ with $(n-2)$ circular arc slits centered at the origin.

D_2: an annulus, $(0<)\ r_1<|z|<r_2\ (<\infty)$ with $(n-2)$ radial slits emanating from the origin.

D_3: the unit circle with $(n-1)$ circular arc slits centered at the origin.

D_4: the unit circle with $(n-1)$ radial slits emanating from the origin.

D_5: the whole plane with n circular arc slits centered at the origin.

D_6: the whole plane with n radial slits emanating from the origin.

2. We shall first quote Hayman's result (p. 33 in [2]).

Lemma. *Let $f(z)=\mathrm{Re}^{i\Phi}$ be single-valued, regular, areally mean p-valent in a domain D and $n(R,\Phi)$ denote the quantity defined above. Let $R_1=\inf_{z\in D}|f(z)|$ and $R_2=\sup_{z\in D}|f(z)|$. Then we have*

$$\int_{R_1}^{R_2} \frac{p(R)}{R} dR \leq p\left(\log \frac{R_2}{R_1}+\frac{1}{2}\right) \tag{2.1}$$
$$\left(p(R)\equiv \frac{1}{2\pi}\int_0^{2\pi} n(R,\Phi)d\Phi\right).$$

Hereafter we shall derive the results in this paper by the method quite similar to [1].