19. *Tail Probabilities for Positive Random Variables Satisfying Some Moment Conditions*

By Norio KÔNO
Institute of Mathematics, Yoshida College, Kyoto University

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1. Let X be a positive random variable such that the asymptotic inequality

$$(c(1-\varepsilon))^{2n}\Gamma(2n+1)^{\beta}\leq E[X^{2n}]\leq(d(1+\varepsilon))^{2n}\Gamma(2n+1)^{\beta} \qquad (n\text{: integer})$$

holds for all ε, $0<\varepsilon<1$, where $0<c\leq d<+\infty$ and $0<\beta<1$. Then L. Davies [1] has proved the following inequalities as a corollary of his theorem:

$$\begin{aligned}\beta d^{-1/\beta}&\leq \varliminf_{x\to+\infty} -\log P(X\geq x)/x^{1/\beta}\\ &\leq \varlimsup_{x\to\infty}-\log P(X\geq x)/x^{1/\beta}\\ &\leq \beta d^{-1/\beta}(r_u/r_l)^{1/\beta},\end{aligned}$$

where $0<r_l\leq 1\leq r_u<+\infty$ are the two positive roots of $f(y)=0$,

$$f(y)=\beta(c/d)^{1/\beta}y^{1/\beta}/(1-\beta)-y/(1-\beta)+1.$$

We will extend his result to a class of positive random variables satisfying some moment conditions which includes his result. For this aim, we shall define "nearly regularly varying function with index α" which is first introduced in [2].

2. Let $\sigma(x)$ be a positive measurable function defined on $[c_0+\infty)$, $(c_0>0)$. We say that $\sigma(x)$ is a "nearly regularly varying function with index α" if and only if there exist two positive constants $r_1\geq r_2$ and a slowly varying function $s(x)$ such that

$$r_2x^{\alpha}s(x)\leq\alpha(x)\leq r_1x^{\alpha}s(x).$$

In particular, we say that $\sigma(x)$ is a "nearly slowly varying function" if $\alpha=0$.

As is well known (for example see [3]) $s(x)$ is represented as follows:

$$s(x)=b(x)\exp\int_{\cdot}^{x}a(t)/t\,dt,$$

where $a(x)$ and $b(x)$ are measurable functions such that

$$\lim_{x\to\infty}b(x)=b>0 \quad\text{and}\quad \lim_{x\to\infty}a(x)=0.$$

3. Theorem 1. *Let X be a positive random variable. Assume that there exist two positive constants c_1 and h, and also a non-decreasing nearly regularly varying function $\sigma(x)$ with index α. $0<\alpha<1$, defined on $[1/h,+\infty)$ such that*