18. *Weak Stability Implies Structural Stability under Axiom A*[*)]

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§ 1. **Introduction.** Let $\mathcal{D}^r(M)$ and $\mathcal{X}^r(M)$ be the sets of diffeomorphisms and vector fields, respectively, of class C^r on a smooth manifold M with Whitney C^r topology. $f \in \mathcal{D}^r(M)$ or $X \in \mathcal{X}^r(M)$ is said to be *weakly C^r stable* if and only if there is a neighborhood N of f or X in $\mathcal{D}^r(M)$ or $\mathcal{X}^r(M)$ such that for any g or Y in N the set of all elements topologically equivalent to g or Y is dense in N, respectively.

In [2] and [3], weak stability researched. The set of all dynamical systems on M which are weakly stable but not structurally stable is an open subset of $\mathcal{D}^r(M)$ or $\mathcal{X}^r(M)$.

Theorem 1 ([2]). *For $1 \leqq r \leqq \infty$ there exists an open set N of $\mathcal{X}^r(\boldsymbol{R}^2)$ such that* (i) *N contains uncountably many equivalence classes of vector fields*; (ii) *for any X in N the set of all elements topologically equivalent to X is dense in N, hence, any X in N is not structurally C^r stable but is weakly C^r stable*; *and* (iii) *any X in N is $C^r\Omega$-stable.*

Theorem 2 ([3]). *Let f be a C^r diffeomorphism of a compact manifold, $1 \leqq r \leqq \infty$. If f is weakly C^r stable, then all periodic points of f are hyperbolic.*

The following problem is open yet: Are there diffeomorphisms or flows on a compact manifold which are weakly stable but not structurally stable? Having in view of this problem, we ask when weak stability implies structural stability. This paper is motivated by this question. There is the following result about this question.

Theorem 3 ([2]). *For a compact manifold M, let f be a C^r diffeomorphism on M, $r \geqq 1$. Suppose that $\Omega(f)$ is a finite set. Then, if f is weakly C^r stable f is structurally C^r stable.*

The main result of this paper is Theorem in the next section.

§ 2. **Result for $\mathcal{D}^r(M)$.** The following Proposition can be said to be an extension of Theorem 3, for Morse-Smale diffeomorphisms are structurally stable.

Proposition. *Let M be compact and $1 \leqq r \leqq \infty$. Then, if $f \in \mathcal{D}^r(M)$ is weakly C^r stable, f is Kupka-Smale.*

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*) Dedicated to Professor Ryoji Shizuma on his 60-th birthday.