17. Cohomology of the Symmetric Space EI

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§ 0. **Introduction.** Among the compact 1-connected irreducible symmetric spaces of exceptional type, $\boldsymbol{G}, \boldsymbol{FI}, \boldsymbol{EI}, \boldsymbol{EII}, \boldsymbol{EV}, \boldsymbol{EVI}, \boldsymbol{EVIII}$ and $\boldsymbol{EIX}$ have 2-torsion. The cohomology of $\boldsymbol{G}, \boldsymbol{FI}$ and $\boldsymbol{EII}$ are known [2], [4], [5]. In this paper we determine the cohomology of $\boldsymbol{EI}=E_6/PSp(4)$.

Since $\boldsymbol{EI}$ is 1-connected and $\pi_2(\boldsymbol{EI})=\boldsymbol{Z}_2$, we have two fiberings

(a) $\widetilde{\boldsymbol{EI}} \xrightarrow{g} \boldsymbol{EI} \xrightarrow{f} K(\boldsymbol{Z}_2\,;2)$,

(b) $E_6 \longrightarrow \widetilde{\boldsymbol{EI}} \longrightarrow BSp(4)$.

On the other hand $\boldsymbol{EIV}=E_6/F_4$, and the subgroups F_4 and $PSp(4)$ of E_6 contain the subgroup $U=S^3\cdot Sp(3)$ in common. Noticing that $F_4/U=\boldsymbol{FI}$ and $PSp(4)/U=\boldsymbol{HP}^3$, we have two more:

(c) $\boldsymbol{HP}^3 \xrightarrow{i} E_6/U \xrightarrow{p} \boldsymbol{EI}$,

(d) $\boldsymbol{FI} \longrightarrow E_6/U \longrightarrow \boldsymbol{EIV}$.

We calculate the Serre spectral sequence associated to these fiberings.

Throughout the paper we use the following notations (A being a ring):

$A\{x_i\}=\oplus A\cdot x_i$ and $\Delta(x_i)=A\{m\,; m$ is a simple monomial in $x_i\}$.

Then our results are

Theorem 1. *$H^*(\boldsymbol{EI}\,;\boldsymbol{Z}_2)=\boldsymbol{Z}_2[x_i\,;i=2,3,5,9,11,13,15,17,21,23]/I$, where $Sq^i x_{i+1}=x_{2i+1}$ $(i=1,2,4,8)$, $Sq^j x_{j+7}=x_{2j+7}$ $(j=4,8)$, $Sq^8 x_{13}=x_{21}$ and I is the ideal generated by the following elements* $(x_5'=x_5+x_2x_3)$: $x_2^3+x_3^2$, $x_2^2x_i$ $(i\neq 2,13,21)$, x_j^2 $(j\neq 2,3)$, $x_5'x_9x_{17}$; $x_3x_{13}+x_5'x_{11}$, $x_5'x_{13}$, $x_9x_{13}+x_5'x_{17}$, $x_{17}x_{13}$, $x_3x_{21}+x_9x_{15}$, $x_5'x_{21}+x_9x_{17}$, x_9x_{21}, $x_{17}x_{21}$; $x_{17}x_{11}+x_5'x_{23}$, $x_5'x_{15}+x_3x_{17}+x_9x_{11}$, $x_{17}x_{15}+x_9x_{23}$, $x_{17}x_{23}+x_3x_5'x_9x_{23}$; $x_{11}x_{13}$, $x_{11}x_{15}+x_3x_{23}$, $x_{11}x_{21}+x_9x_{23}$, $x_{13}x_{15}+x_5'x_{23}$, $x_{15}x_{21}$, x_kx_{23} $(k=11,13,15,21)$.

Theorem 2. (i) *As a ring $H^*(\boldsymbol{EI})/\mathrm{Tors.}\,H^*(\boldsymbol{EI})$ is generated by $\{e_i, e_j'\,; i=8,9,17\,; j=16,17,25,34\}$ and*

$$H^*\left(\boldsymbol{EI}\,;\boldsymbol{Z}\left[\frac{1}{2}\right]\right)=\boldsymbol{Z}\left[\frac{1}{2}\right][e_8]/(e_8^3)\otimes\Lambda(e_9,e_{17}),$$

in which $e_{16}'=\frac{1}{4}e_8^2$, $e_{17}'=\frac{1}{2}e_8e_9$, $e_{25}'=\frac{1}{2}e_8e_{17}$ *and* $e_{34}'=\frac{1}{4}e_8e_9e_{17}$.

(ii) *There exist torsion elements $\chi\in H^3(\boldsymbol{EI})$ of order 2 and $\omega_i\in H^i(\boldsymbol{EI})$ $(i=5,11,15,23)$ of order 4, and*