16. *On the Periods of Enriques Surfaces. II*

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This is a continuation of [4], and here we report on our result on the image of the period map for Enriques surfaces.

Let S be an Enriques surface defined over the field of complex numbers. Then there corresponds to S a point $\lambda(S)$, called the period of S, which is in the period space D/Γ. First we recall the construction of D and Γ. Let T be the universal covering of S. Then T is a $K3$ surface, and hence the homology group $H_2(T, \boldsymbol{Z})$, given with the intersection product, is isomorphic to a unique even unimodular euclidean lattice Λ of signature (3, 19). Moreover, if we associate the involution τ induced by the covering transformation, the pair $(H_2(T, \boldsymbol{Z}), \tau)$ is isomorphic to a standard pair (Λ, ρ) (see [4], § 3). Let $\Lambda(-1)$ denote the (-1)-eigenspace of ρ. Then D consists of non-zero linear maps $\omega: \Lambda(-1)\to\boldsymbol{C}$, modulo multiplications by constants, which satisfy the Riemann bilinear relations

$$\omega\cdot\omega=0, \qquad \omega\cdot\bar{\omega}>0,$$

the product being induced by that on $\Lambda(-1)$. On the other hand, Γ is the group of those automorphisms of $\Lambda(-1)$ which are the restrictions of the automorphisms of Λ commuting with ρ.

An element e of $\Lambda(-1)$ is called a *root* if it satisfies $e^2=-2$. From the explicit description of $\Lambda(-1)$ in [4], we infer that such elements exist. If e is a root, we define a hypersurface H_e of D by the condition $\omega(e)=0$. We shall use H_e/Γ to denote $H_e\Gamma/\Gamma$.

Main Theorem. *There exists only a finite number of Γ-equivalence classes of the roots e in $\Lambda(-1)$, and if λ is a point of D/Γ outside of the union of the hypersurfaces H_e/Γ, then λ is the period of an Enriques surface S, which is uniquely determined by λ. Moreover, any point of H_e/Γ is not the period of an Enriques surface.*

The basic idea of the proof is that of [3].

First, by the construction in [4], each Enriques surface S is birationally equivalent to a double covering of $\boldsymbol{P}^1\times\boldsymbol{P}^1$. We take a system of 2-way homogeneous coordinates $(Y_1, Y_2; Z_1, Z_2)$ and fix the projection onto the second factor. Then the branch locus of the covering consists of the two fibres Γ_i defined by $Z_i=0$, $i=1, 2$, and a curve B_E^0 of bidegree (4, 4), which has two 2-fold double points at P_i on Γ_i, having the contact of order 4 with Γ_i at P_i, $i=1, 2$. An Enriques surface S, with an elliptic