12. *On the Divisibility by 2 of the Eigenvalues of Hecke Operators*

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Introduction. Prof. J.-P. Serre presented to us the following problem in his seminar which was held at the Research Institute for Mathematical Science of Kyoto University on the 27th of March in 1976 after the International Symposium on Algebraic Number Theory.

"Let $w+2$ be an even integer $\geqq 2$ and λ_p an eigenvalue of the Hecke operator $T(p)$ on cusp forms of weight $w+2$ for $SL(2, Z)$ where p is a rational prime. It is true that $\lambda_p/2$ is an algebraic integer? (It is true for $w+2 \leqq 96$ and all prime p.)"

In this paper we will give an affirmative answer for the above question. I continue my research in this direction now. I wish to express my gratitude heartily to Prof. J.-P. Serre for presenting this problem and informing me of many kind valuable suggestions on it and on the improvement of this paper.

§ 1. We denote by S_{w+2} the space of cusp forms of weight $w+2$ for the full modular group $SL(2, Z)$. We put $S_{w+2}^R = \{f \in S_{w+2} \mid \text{all the Fourier coefficients of } f \text{ at } z = i\infty \text{ are real numbers}\}$.

Theorem 1. *Any eigenvalue is divisible by* 2 *of the Hecke operator* $T(p)$ *on* S_{w+2} *for any rational prime* p *and any even weight* $w+2$.

Prof. J.-P. Serre informed me two statements equivalent to this theorem.

a) *In the space of cusp forms of any even weight* (*with real or rational coefficients*) *there is a lattice which is stable by the* $T(p)/2$. (*Such a lattice is not unique. But we can describe it nicely. See the proposition below.*)

b) *If* $a_{i,k}(p)$ *denotes the i-th coefficient of the characteristic polynomial of the* $T(p)$ *on* S_k, *we have* $2^i \mid a_{i,k}(p)$ *for any prime* p *and any even weight* k. (*We note that* $a_{i,k}(p)$ *is an integer which can easily be computed on machine.*)

Remark. We have extended this theorem to the case of real cusp forms of weight $w+2$ for some congruence subgroups (see [9], [10]).

Suggested by Prof. Serre, using the results obtained in this paper, I proved a more precise statement, the next Theorem 2, quite recently.

Theorem 2. *On cusp forms for* $SL(2, Z)$, *we have*

(i) *Any eigenvalue of the* $T(p)$ *is divisible by* 4 *for any prime* p *such that* $p \equiv -1 \bmod 4$ *and any weight* $w+2$.