11. *On the Acyclicity of Free Cobar Constructions.* I

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1. Let Λ be a group ring or an enveloping algebra of Lie algebras over $\boldsymbol{Z}$ with an augmentation $\varepsilon : \Lambda \rightarrow \boldsymbol{Z} \rightarrow (0)$. Let (X, ∂) be a complex of left Λ-modules

$$\text{(1.1)} \qquad \longrightarrow X_p \xrightarrow{\partial_p} X_{p-1} \xrightarrow{\partial_{p-1}} \cdots \longrightarrow X_1 \xrightarrow{\partial_1} \Lambda \xrightarrow{\varepsilon} \boldsymbol{Z} \longrightarrow (0)$$

where each X_p is a free left Λ-module and each ∂_p is a left Λ-module homomorphism. Let Λ^f be a free associative algebra over $\boldsymbol{Z}$ such that we get an exact sequence

$$\text{(1.2)} \qquad (0) \longrightarrow L \xrightarrow{\iota_0} \Lambda^f \xrightarrow{\kappa_0} \Lambda \longrightarrow (0)$$

where L denotes an ideal of Λ^f. First we assume

Assumption 1. i) *There exist two sequences* (X^f, ∂^f) *and* $(L \otimes_{\Lambda^f} X^f, 1 \otimes \partial^f)$ *of left* Λ^f*-modules on the augmentation* ε^f,

$$\text{(1.3)} \qquad (0) \longrightarrow (\Lambda^f)^+ \longrightarrow \Lambda^f \xrightarrow{\varepsilon^f} \boldsymbol{Z} \longrightarrow (0)$$

such that the following diagram commutes:

$$\begin{array}{ccccccccccc}
 & (0) & & (0) & & & & (0) & & (0) & \\
 & \uparrow & & \uparrow & & & & \uparrow & & \uparrow & \\
\longrightarrow & X_p & \xrightarrow{\partial_p} & X_{p-1} & \longrightarrow & \cdots & \longrightarrow & X_1 & \xrightarrow{\partial_1} & \Lambda & \longrightarrow \boldsymbol{Z} \longrightarrow (0) \\
 & \uparrow \kappa_p & & \uparrow \kappa_{p-1} & & & & \uparrow \kappa_1 & & \uparrow \kappa_0 & \\
\longrightarrow & X_p^f & \xrightarrow{\partial_p^f} & X_{p-1}^f & \longrightarrow & \cdots & \longrightarrow & X_1^f & \xrightarrow{\partial_1^f} & \Lambda^f & \longrightarrow \boldsymbol{Z} \longrightarrow (0) \\
 & \uparrow \iota_p & & \uparrow \iota_{p-1} & & & & \uparrow \iota_1 & & \uparrow \iota_0 & \\
\longrightarrow & L \otimes_{\Lambda^f} X_p^f & \xrightarrow{1 \otimes \partial_p^f} & L \otimes_{\Lambda^f} X_{p-1}^f & \longrightarrow & \cdots & \longrightarrow & L \otimes_{\Lambda^f} X_1^f & \xrightarrow{1 \otimes \partial_1^f} & L & \\
 & \uparrow & & \uparrow & & & & \uparrow & & \uparrow & \\
 & (0) & & (0) & & & & (0) & & (0) &
\end{array}$$

where each κ_p *denotes an epimorphism as left modules and each* X_p *or* X_p^f *is isomorphic to the tensor product* $\Lambda \otimes S_p$ *or* $\Lambda^f \otimes S_p$ *respectively for a free abelian group* S_p *such that* $L = X_2^f \otimes \Lambda^f$.

ii) ∂_p^f *are all injective on* X_p^f, $p \geqq 1$.

Let $\overset{+}{X}{}^f$ be the direct sum $\bigoplus_{p=2}^{\infty} X_p^f \oplus (\Lambda^f)^+$ and $T(\overset{+}{X}{}^f)$ be the tensor algebra of $\overset{+}{X}{}^f$ denoted by A. A becomes a free graded algebra $\bigoplus_{s=0}^{\infty} A_s$, where A_s is spanned by the elements of the form $u_1 u_2 \cdots u_{m-1} u_m$, $u_j \in X_{p_j}^f$ $(p_j \geqq 2)$, $j \leqq m-1$, and $u_m \in \Lambda^f$ such that $s = \sum_{j=1}^{m-1} (p_j - 1)$.

We introduce a lexicographic order into a basis of A: Let $\{u_\gamma^p, \gamma \in \Gamma_p\}$ be an ordered basis of S_{p+1}, $p \geqq 1$ and $\{u_\gamma^0, \gamma \in \Gamma_0\}$ be an ordered system