54. On the 2-Components of the Unstable Homotopy Groups of Spheres. I

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(Communicated by Kunihiko KODAIRA, M. J. A., Nov. 12, 1977)

The 2-components π_{n+i}^n of the homotopy groups of spheres $\pi_{n+i}(S^n)$ have been determined in the unstable range for all n when $i \leqq 24$ [4, 6, 7, 11]. The purpose of this note is to summarize briefly the results on the unstable homotopy groups of spheres which have been obtained as the application of the composition method of H. Toda [11]. Making use of the generators given in [2, 4–9, 11] and the new ones defined in this note, we will state, in this part I, the results on the 2-components π_{n+i}^n for all n*) when $25 \leqq i \leqq 28$. Further results will be stated in the part II.

The results overlap in the metastable range with those of M. Mahowald [3] and S. Thomeier [10].

1. On the 25-stem. *There are following new elements*: $\phi' \in \pi_{28}^3$, $\phi'' \in \pi_{30}^5$ *and* $\phi''' \in \pi_{32}^7$ *with the Hopf invariants* ϕ_5 (*mod* $4\nu_5 \circ \bar{\kappa}_8, \alpha_2'$), σ_9^3 *and* $\bar{\sigma}_{13}$ *respectively*.

Now, for an element $\alpha \in \{\eta_4, 2\iota_5, \phi_5\}$, we have $2\alpha \equiv \eta_4 \circ \delta_5$ (*mod* $\eta_4 \circ \bar{\mu}_5 \circ \sigma_{22}, E\nu' \circ \varepsilon_7 \circ \kappa_{15}$). Hence we conclude that *there exists an element* ϕ' *such that* $2\phi' \equiv \eta_3 \circ \delta_4$ (*mod* $\eta_3 \circ \bar{\mu}_4 \circ \sigma_{21}$) *and* $E\phi' \in \{\eta_4, 2\iota_5, \phi_5\}$ (*mod* $\nu_4 \cdot \pi_{29}^7$). This implies

$$\pi_{28}^3 = Z_4\{\phi'\} \oplus Z_2\{\nu' \circ \varepsilon_6 \circ \kappa_{14}\} \oplus Z_2 \oplus Z_2.$$

In the above group and the following ones, *the last two direct summands* $Z_2 \oplus Z_2$ *stand for* $Z_2\{\mu_{3,n}\} \oplus Z_2\{\eta_n \circ \bar{\mu}_{n+1} \circ \sigma_{n+18}\}$ $(n \geqq 3)$ *which survive in the stable range*. Next, the relation $2\phi'' \equiv \pm E^2\phi'$ (*mod other elements*) holds for an element $\phi'' \in \{\nu_5, E\sigma' \circ \sigma_{15}, \sigma_{22}\}_1$ and we see

$$\pi_{30}^5 = Z_8\{\phi''\} \oplus Z_2\{\nu_5 \circ \varepsilon_8 \circ \kappa_{16}\} \oplus Z_2\{\nu_5^2 \circ \bar{\sigma}_{11}\} \oplus Z_2 \oplus Z_2,$$

$$\pi_{31}^6 = Z_4\{\Delta(\bar{\kappa}_{13})\} \oplus Z_2\{\Delta(EA_1 \circ \varepsilon_{25})\} \oplus Z_2\{\Delta(EA_1^{(1)})\} \oplus Z_8\{E\phi''\} \oplus Z_2\{\nu_6^2 \circ \bar{\sigma}_{12}\} \oplus Z_2 \oplus Z_2$$

Let us choose $\phi''' \in \{\sigma' \circ \sigma_{14}, \sigma_{21}, \nu_{28}\}_1$. Since $\nu_7^2 \circ \bar{\sigma}_{13} \in 2\pi_{32}^7$, it follows that $2\phi''' \equiv \nu_7^2 \circ \bar{\sigma}_{13}$ (*mod* $2E^2\phi''$), which determines

$$\pi_{32}^7 = (Z_4 \oplus Z_8)\{\phi''', E^2\phi''\} \oplus Z_2\{\sigma' \circ \eta_{14} \circ \bar{\mu}_{15}\} \oplus Z_2 \oplus Z_2.$$

Making use of the relations $E^2\phi''' \equiv \sigma_9 \circ \nu_{16}^*$ (*mod* $\nu_9^2 \circ \bar{\sigma}_{15}$) *and* $\sigma' \circ \xi_{14} \equiv E^2\phi''$ (*mod* $2E\phi''', 2E^2\phi'', \eta_7 \circ \bar{\mu}_8 \circ \sigma_{25}$), we obtain

$$\pi_{34}^9 = Z_8\{\sigma_9 \circ \xi_{16}\} \oplus Z_2\{\sigma_9 \circ \eta_{16} \circ \bar{\mu}_{17}\} \oplus Z_4\{\sigma_9 \circ \nu_{16}^*\} \oplus Z_2 \oplus Z_2,$$

*) We omit the cases that $n=2$, 4 and 8; the results are immediate from Proposition 4.4 of [11].