

43. *Three-Dimensional Dirichlet Problem for the Helmholtz Equation for an Open Boundary*

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(Communicated by Kôzaku YOSIDA, M. J. A., Oct. 12, 1977)

1. Let us mean by an open boundary a union of a finite number of simple, smooth, bounded, simply or multiply connected, and two-sided open surfaces in R^3 , each of which being bounded by a union of piecewise smooth, simple and closed contours of finite length. It is the purpose of this paper to publish the résumé of the theory of the Dirichlet problem for the Helmholtz equation and for an open boundary which the author has established recently.

In the previous papers [1, 2], the author completed the theory of the two-dimensional Dirichlet problem for an open boundary composed of a union of simple and smooth arcs in a plane. In [1], he depended on a theory of a singular integral equation, whose approach was difficult to extend to apply to the three-dimensional case. However, the series expansion approach taken in [2] was studied carefully so that it can be extended to apply to the three-dimensional problems. The extension will be described in the present paper.

Since the space does not allow a detailed description, only the main results will be given, and the full length paper is expected to appear in some periodical soon.

2. Let S be the above mentioned union of open surfaces, ∂S be its periphery, and set $S^0 = S - \partial S$. Let us denote points in R^3 by x, y , etc., the distance between x and y by $d(x, y)$, and the distance between x and ∂S by $d(x, \partial S)$. Suppose that $S(\rho)$ and $S^*(\rho)$, where ρ is a positive constant, are defined by $S^*(\rho) = \{x; x \in S, d(x, \partial S) < \rho\}$ and $S(\rho) = S - S^*(\rho)$, respectively. Assume that the unit vector normal to S is given on S^0 . With regards to the directions of the normal, each side of S is called the positive or negative side of it, respectively. If a function $f(x)$ assumes a definite limit as $x (\in R^3 - S)$ tends to a point x_0 on S_0 from the positive (negative) side of S , it is said to be continuous on S from the positive (negative) side of it, and the limiting value is denoted by $f^+(x)(f^-(x))$. Let $C(S)$, $L_1(S)$ and $L_2(S)$ be, as usual, the spaces of functions which are continuous on S , integrable on S , and square integrable on S , respectively. Suppose that $T(S)$ is the set of all functions belonging to $L_1(S) \cap C(S(\rho))$ for any $\rho > 0$. Our problem, which will be called Problem D for the sake of brevity, is to find a function $u(x)$