4. *Topologically Unequivalent Diffeomorphisms Whose Suspensions Are C^∞ Equivalent*

By Gikō IKEGAMI
Department of Mathematics, College of General Education, Nagoya University

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Let $\psi_t(x,s)=(x,s+t)$ be the trivial flow on $M\times \boldsymbol{R}$. Let $M_f=M\times \boldsymbol{R}/(f(x),t)\sim(x,t+1)$ be the attaching torus of a diffeomorphism f on M. The flow φ_t on M_f induced by ψ_t is called a *suspension* of f.

If two diffeomorphisms f and f' on M and M', respectively, are C^r equivalent (C^r conjugate) the suspensions φ and φ' of f and f' are C^r equivalent; i.e. there is a C^r diffeomorphism from M_f to $M'_{f'}$ mapping any orbit of φ onto an orbit of φ' with preserving the orientations of orbits. But the converse is not true. (See [1] or [2].) In case that there is no surjection $\pi_1(M)\to \boldsymbol{Z}$ or $\pi_1(M')\to \boldsymbol{Z}$, the C^r equivalence of φ and φ' implies the C^r equivalence of f and f'. (See [1].)

M. M. Peixoto asked to the author whether there exist topologically unequivalent two diffeomorphisms on the same manifold whose suspensions are equivalent. Next theorem was motivated by this question.

Theorem. *Let N be a compact manifold with* $\dim N\geqq 0$ *and let* $M=N\times S^1$, *where S^1 is the* 1-*sphere. Then, there are infinitely many Morse-Smale C^∞ diffeomorphisms f_i $(i=1,2,\cdots)$ on M satisfying the following properties.*

i) *The all suspensions of f_i $(i=1,2,\cdots)$ are C^∞ equivalent.*

ii) *If $i\neq j$, f_i and f_j are not topologically equivalent.*

Lemma. *Let f be a diffeomorphism on $M=N\times S^1$ with at least one periodic point such that f is diffeotopic to the identity. (i.e. there is a smooth map $F:M\times I\to M$ such that $F(\ ,0)=id.$, $F(\ ,1)=f$, and that $F(\ ,t)$ is a diffeomorphisms on M for any $t\in I$, where $I=[0,1]$.) Then there are C^∞ diffeomorphisms f_i $(i=1,2,\cdots)$ satisfying the following properties.*

i) $f_1=f$.

ii) *The all suspensions of f_i $(i=1,2,\cdots)$ are C^∞-equivalent.*

iii) *If $i\neq j$, f_i and f_j are not topologically equivalent.*

Proof. Since f is deffeotopic to the identity, M_f is diffeomorphic to $M\times S^1$. We may consider the suspension φ_t of f as a flow on $M\times S^1$ such that for any $s\in S^1 M\times s$ is a cross-section of φ_t. We define a submanifold M_n of $M\times S^1$ for $n=2,3,\cdots$ as follows.

$\tilde{M}_n=\{(x,e^{2\pi nti},t)\in N\times S^1\times I\mid x\in N, t\in I\}$ is a codimension one sub-