

70. On the flat conformal differential geometry II.

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§ 2. Canonical fundamental equations of flat conformal geometry.

In the preceding Chapter,¹⁾ we have established the so-called fundamental differential equations for the flat conformal geometry, and discussed the transformation law of the coefficients of the fundamental equations under change of factors and under coordinate transformations respectively. We have also studied the integrability conditions of the fundamental differential equations.

In the present Chapter, we shall introduce a canonical form of the fundamental equations and discuss the transformation law of the coefficients of the canonical fundamental equations under coordinate transformations, the factor being fixed in this case. We shall, in the last Paragraph of the present Chapter, also study the integrability conditions of the canonical fundamental differential equations.

1. Canonical fundamental equations.²⁾

We have seen that, the fundamental equations established for a repère $[A_0, A_\lambda, A_\infty]$ being

$$(2.1) \quad \begin{cases} \frac{\partial A_0}{\partial \xi^\nu} = A_\nu, \\ \frac{\partial A_\mu}{\partial \xi^\nu} = \Pi_{\mu\nu}^0 A_0 + \Pi_{\mu\nu}^\lambda A_\lambda + \Pi_{\mu\nu}^\infty A_\infty, \\ \frac{\partial A_\infty}{\partial \xi^\nu} = \Pi_{\infty\nu}^\lambda A_\lambda, \end{cases}$$

if we perform a change of factor

$$(2.2) \quad {}^*A_0 = \phi A_0,$$

the repère $[A_0, A_\lambda, A_\infty]$ will be transformed into another repère $[{}^*A_0, {}^*A_\lambda, {}^*A_\infty]$ following the formulae

$$(2.3) \quad \begin{cases} {}^*A_0 = \phi A_0, \\ {}^*A_\mu = \phi(\phi_\mu A_0 + A_\mu), \\ {}^*A_\infty = \frac{1}{\phi} \left(\frac{1}{2} \phi^\lambda \phi_\lambda A_0 + \phi^\lambda A_\lambda + A_\infty \right); \end{cases}$$

1) K. Yano: On the flat conformal differential geometry I. Proc., **21** (1945) 419.

2) T. Y. Thomas: On conformal geometry. Proc. Nat. Acad. Sci. U.S.A., **12** (1926), 352-359.