64. *On the flat conformal differential geometry I.*

By Kentaro YANO.

Mathematical Institute, Tokyo Imperial University.

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§ 0. *Introduction.*

The generalization of conformal geometry was first discussed by H. Weyl[1] who considered a conformal transformation of the form $\bar{g}_{\mu\nu}=\rho^2 g_{\mu\nu}$ of the fundamental tensor of a Riemannian space. The theorems which state properties invariant under this transformation of the fundamental tensor constitute the conformal geometry of Riemannian spaces. The conformal geometry of Riemannian spaces was studied, from this point of view, especially by geometers of the American School, A. Fialkow,[2] V. A. Hoyle,[3] J. Levine,[4] V. Modesitt,[5] J. M. Thomas,[6] T. Y. Thomas,[7] J. Vanderslice,[8] O. Veblen[9] and others. The relative tensor $G_{\mu\nu}=g_{\mu\nu}/g^{\frac{1}{n}}$, where g is the determinant formed with components $g_{\mu\nu}$ of the fundamental tensor, being invariant under a conformal transformation, T. Y. Thomas defined the conformal geometry as the theory of invariants of relative quadratic differential form $G_{\mu\nu}d\xi^{\mu}d\xi^{\nu}$.

The study of conformal geometry of Riemannian spaces was continued in

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(3) V. A. Hoyle: Some problems in conformal geometry. Diss. Princeton Univ., (1931).

(4) J. Levine: Conformal-affine connections. Proc. Nat. Acad. Sci. U.S.A., **21** (1935), 165–167; New identities in conformal geometry. Duke Math. Journ., **1** (1935), 173–184; Conformal scalars. Bull. Amer. Math. Soc., **42** (1936), 115–124; Groups of motions in conformally flat spaces. Ibid. 418–422.

(5) V. Modesitt: Some singular properties of conformal transformations between Riemannian spaces. Amer. Journ. Math., **60** (1938), 325–336.

(6) J. M. Thomas: Conformal correspondence of Riemannian spaces. Proc. Nat. Acad. Sci. U.S.A., **11** (1925), 257–259; Conformal invariants. Ibid. **12** (1926), 389–393.

(7) T. Y. Thomas: Invariants of relative quadratic differential forms. Ibid. **11** (1925), 722–725; On conformal geometry. Ibid. **12** (1926), 352–359; Conformal, tensors (First note). Ibid. **18** (1932), 103–112; Conformal tensors (Second note). Ibid. 188–193; The differential invariants of generalized spaces. Cambridge University Press, (1935).

(8) J. Vanderslice: Conformal tensor invariants. Proc. Nat. Acad. Sci. U.S.A., **20** (1934), 672–676.

(9) O. Veblen: Conformal tensors and connections. Ibid. **14** (1928), 735–745; Formalism for conformal geometry. Ibid. **21** (1935), 168–173.