55. *Lie derivatives in general space of paths.**

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§ 0. *Introduction.* In a series of Notes published in these Proceedings,[1] the present author has studied the infinitesimal deformations in affinely connected spaces. The main purpose of the present Note is to study the infinitesimal deformations in the general space of paths and to generalise some of the results obtained in the above cited Notes. To my knowledge, there are only three parers on the infinitesimal deformations in the generalised spaces, the papers by M.S. Knebelman,[2] by S. Hokari[3] and by E. T. Davies.[4]

In Paragraph 1, we expose some formulae in the geometry of general space of paths which will be useful later. In the next Paragraph, we study the Lie derivations of tensors whose components are functions not only of position but also of direction. In Paragraph 3, we shall define the Lie derivatives of the affine connection and study some of its fundamental properties.

In Paragraph 4, we study the affine and projective collineations in the general space of paths which were also studied by M.S. Knebelman.

In the last Paragraph, we define the deformed space whose components of the affine connection are $\Gamma^{\lambda}_{\mu\nu}+D\Gamma^{\lambda}_{\mu\nu}$, and calculate the curvature tensor of the deformed space. The full detail will be published elsewhere.

§ 1. *General space of paths.*[5] A general space of paths is an n-dimensional space in which is given a system of curves, called paths, such that through any two points given in a properly restricted region, there passes one and only one path. If we introduce, in this general space of paths, a system of coordinates

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1) K. Yano: Bemerkungen über infinitesimale Deformationen eines Raumes, Proc., **21** (1945), 171 ; Sur la déformation infinitésimale des sous-espaccs dans un espace affine, ibidem, 248 ; Sur la déformation infinitésimale tangentielle d'un sous-espace, ibidem, 261; Quelques remarques sur un article de M.N. Coburn intitulé " A characterization of Schouten's and Hayden's deformation methods," ibidem, 330.

2) M. S. Knebelman: Collineations and motions in generalized spaces, Amer. Journ. of Math., **51** (1929), 527-564.

3) S. Hokari: Winkeltreue Transformationen und Bewegungen im Finslerschen Raume, Journal of the Faculty of Science, Hokkaido Imp. Univ., **5** (1936), 1-8.

4) E. T. Davies: Lie derivation in generalized metric space, Annali di Mat., **18** (1939), 261-274.

5) J. Douglas: The general geometry of paths, Annals of Math., **29** (1928), 143-168.