# 18. *Fundamental Theory of Toothed Gearing (III).*

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Suppose that a pair of pitch curves $K_1$ and $K_2$ and a pair of profile curves $F_1$ and $F_2$ invariably connected with $K_1$ and $K_2$ be given. Let $P$ be a common pitch point at a certain instant, $C$ be the point of contact of $F_1$ and $F_2$ corresponding to $P$. Suppose that after infinitesimal time interval $dt$ two points $P_1$ and $P_2$ on $K_1$ and $K_2$ and two points $C_1$ and $C_2$ on $F_1$ and $F_2$ may respectively come to the point of contact. Denote by $ds$ the length of the arc $PP_1$, and consequently that of $PP_2$, and by $dp_1$ and $dp_2$ the lengths of the arcs $CC_1$ and $CC_2$ respectively. The pitch curve $K$ is oriented and accordingly $ds$ has a sign positive or negative. We shall give also a sign to $dp$; $dp$ is positive or negative according as the part of arc $dp$ of the profile curve $F$ is of positive or negative type.

§ 1. Sliding of profile curves.

At the sliding contact motion of $F_1$ and $F_2$ during the time $dt$ the point $C$ on $F_1$ slides along $F_2$ for the distance $dp_2-dp_1$, and consequently its velocity $v_{p1}$ is given by

$$v_{p1}=\frac{dp_2-dp_1}{dt}. \tag{1}_1$$

$v_{p1}$ is named the velocity of sliding of $F_1$ (at the point $C$ on $F_2$). In like manner the velocity of sliding of $F_2$ may be defined:

$$v_{p2}=\frac{dp_1-dp_2}{dt}. \tag{1}_2$$

Evidently $v_{p1}$ and $v_{p2}$ have the same absolute value and the different signs.

Denoted by $\omega_1$ and $\omega_2$ respectively the instant angular velocities of $K_1$ and $K_2$ at their rolling contact motions, and we say that the sign of the angular velocity $\omega$ is positive or negative according as $K$ rotates clockwise or counter-clockwise. Denoting by $a_1$ and $a_2$ the radii of curvature of $K_1$ and $K_2$ respectively at the instant common pitch point $P$ we have

$$\omega_1=\frac{1}{a_1}\frac{ds}{dt}, \qquad \omega_2=\frac{1}{a_2}\frac{ds}{dt}. \tag{2}$$

Let $\omega$ denote the relative rolling angular velocity of $K_1$ to $K_2$, then obviously $\omega=\omega_1-\omega_2$ and accordingly from (2)