4. A Lattice-Theoretic Treatment of Measures and Integrals.

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In this paper we shall introduce a system of mathematical objects which is considered as a generalization of a set of Somen introduced by C. Carathéodory in his article [2], and which contains as particular cases a system of subsets of a set and a system of non-negative functions defined on a set. We shall give further the results corresponding to the theory of the Carathéodory's outer measure [1] and the extension theorem of Kolomogoroff-Hopf. [2]

1. In this section we shall deal with a mathematical object $\mathfrak{M}$ satisfying the following axioms.

Axiom 1. For every $A, B \in \mathfrak{M}$, one of the incompatible formulas $A=B$ or $A \neq B$ is accepted and "$=$" satisfies the following conditions:

(1.1) $A=A$; (1.2) If $A=B$, then $B=A$; (1.3) If $A=B$ and $B=C$, then $A=C$.

Axiom 2. For every $A, B \in \mathfrak{M}$, there exists only one element of $\mathfrak{M}$ denoted by $A \dot{+} B$, satisfying the following conditions:

(2.1) $A \dot{+} A=A$; (2.2) $A \dot{+} B=B \dot{+} A$; (2.3) $A \dot{+} (B \dot{+} C)=(A \dot{+} B) \dot{+} C$; (2.4) If $B=B'$, then $AB \dot{+} =A \dot{+} B'$.

$A \dot{+} B$ will be called the *sum* of A and B.

Definition 1. If $A \dot{+} B=A$, then B is said to be a *part* of A and denoted by $A \supseteq B$ or $B \subseteq A$.

Then $\mathfrak{M}$ may be regarded as an *ordered system* through the relation $A \supseteq B$ which we can replace by $A \geqq B$ and in this case Definition 1 should be taken as the definition of the enouncement "B is smaller than A" or "A is greater than B".

Axiom 3. For $\{A_n\}$, $A_n \in \mathfrak{M}$ [3], there exists the smallest element $V \in \mathfrak{M}$, of which every A_n is a part, and it will be written $V=A_1 \dot{+} A_2 \dot{+} \dots$ or $V=\sum_{n=1}^{\infty} A_n$. V will be called the *sum* of $\{A_n\}$.

Axiom 4. There exists an element of $\mathfrak{M}$ which is a part of every element of $\mathfrak{M}$ and is called a *null element*.

Definition 2. For $A, B \in \mathfrak{M}$, if A and B has no common part except the null element, then we say that A and B are *disjunct* and write $A \circ B$ or $B \circ A$.

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1) Cf. [1].

2) Cf. [1].

3) A_n denotes A_n $(n=1, 2, \dots)$.